The surprising relevance of a continuum description to granular clusters

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Nature shuns homogeneity. In turbulent clouds, industrial reactors and geophysical flows, discrete particles arrange in clusters, posing difficult challenges to theory. A persistent question is whether clusters can be modeled with continuum equations. Recent evidence indicates that suitable equations can predict the formation of clusters in granular flows, despite violating the simplifying assumptions upon which they are based.

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1. Introduction

The article by Mitrano *et al.* (2013) addresses the pivotal question whether particle suspensions can be modeled by continuum equations, despite the inevitable formation of clusters. Whether particles interact with atmospheric turbulence as cloud aerosols (Chun *et al.* 2005), flow as a suspension through chemical reactors (Pepiot & Desjardins 2012), or rise in volcanic eruptions (Doronzo *et al.* 2012), they invariably produce clusters that theory cannot ignore. Industrial risers, for example, lift catalyst particles with a gas to stimulate chemical reactions. A challenge is to predict the drag on particles, so their concentration and reaction rates may be calculated. Unfortunately, the average drag force on clustered solids is much less than if particles were homogeneously distributed (Helland *et al.* 2007), a fact that wild geese exploit to fly long distances.

An essential ingredient for cluster formation is inertia, which compels particles to skip fluid streamlines (Maxey 1987). Kinetic energy losses exacerbate this mechanism, either as particles interact with the surrounding fluid (Wylie & Koch 2000), or as they collide inelastically with one another (Conway & Glasser 2004). Turbulence also congregates particles in regions where their collisions are more frequent than if they were homogeneously distributed (Longmire & Eaton 1992; Duncan *et al.* 2005; Simonin *et al.* 2006; Pan & Padoan 2010; Bec *et al.* 2010).

Numerical simulations reveal details of cluster dynamics by tracking individual particles, either as points subject to a local drag law (van der Hoef *et al.* 2008; Capecelatro & Desjardins 2013), or by resolving the flow around them (Wylie *et al.* 2003). However, real systems can involve more particles than numerical simulations can directly handle. For example, a riser of 1 m diameter and 20 m height holding a mere 0.2% solids by volume already contains $\sim 10^{11}$ particles of 70 μm.

Treating particles as a continuum allows computation on a larger scale (Agrawal *et al.* 2001; Fox 2012). Yet, a persistent question is whether this can capture cluster formation, particularly when particles cross (Desjardins *et al.* 2008). Therefore, because clusters also form in a granular "gas" of inelastic particles colliding in a vacuum (Hopkins *et al.*

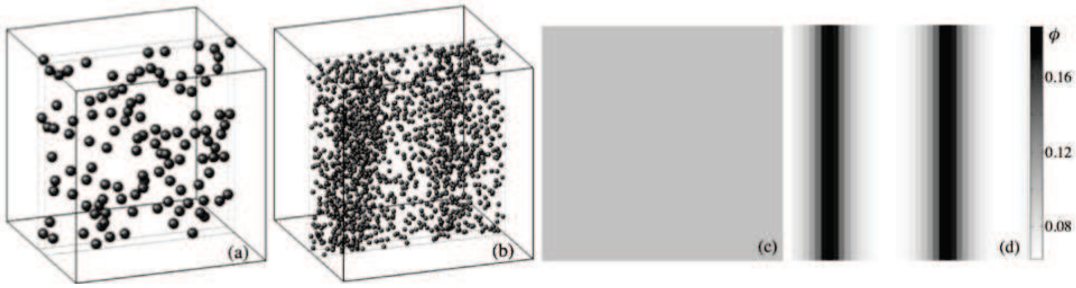

FIGURE 1. Mitrano *et al.* (2013) juxtapose (a,b) simulations and (c,d) solutions of the equations of Garzó & Dufty (1999) having similar features. For a large enough domain, clusters form in simulations (b), and they also arise as instabilities of the continuum theory (d).

1993; McNamara 1993), it is instructive to test, as Mitrano *et al.* (2013) did, whether hydrodynamic equations are relevant to the particle phase without a fluid.

2. Overview

Hydrodynamic equations for granular gases are obtained by extending the Maxwell-Boltzmann kinetic theory to inelastic impacts. A central concept is the granular temperature which, as with a gas of hard spheres, represents the kinetic energy stored in velocity fluctuations. At first, Jenkins & Richman (1988) and others considered weak collisional energy dissipation, mostly with smooth inelastic spheres or disks, but possibly involving frictional interactions as well (Lun 1991; Jenkins & Zhang 2002). The resulting equations reproduced most features of numerical simulations and experiments without gravity (Xu *et al.* 2003, 2009), but they did not apply to highly inelastic systems.

To derive a granular hydrodynamics for higher collisional energy dissipation, Garzó & Dufty (1999) considered a homogeneous cooling process, in which particles steadily lose their initial agitation through collisions, without any mechanism to replenish it. Their calculations uncovered concentration gradient terms in the energy flux that become important at high inelasticity. However, as Goldhirsch & Zanetti (1993) had observed in molecular dynamics (MD) simulations, the cooling process readily forms clusters, seemingly contradicting the suspension homogeneity that Garzó & Dufty (1999) invoked.

At first glance, particle clusters also appear to challenge two chief simplifications of the kinetic theory. In the first, particles are required to forget their past rapidly, so the theory can easily handle reshuffling of their statistical velocity distribution after impacts. Long-lasting coherent clusters would seem to cast doubt on this "molecular chaos" assumption.

In the second simplification, most kinetic theories forbid variations of flow variables on a scale less than the mean free path between consecutive collisions, so the granular gas is not rarefied. In other words, the Knudsen ratio Kn of mean free path to a gradient length scale is taken to be small. Yet, with a periphery featuring steep variations in flow variables next to relatively small concentrations, clusters can produce regions of high mean free paths and large Kn based on velocity gradients.

In principle, the Navier-Stokes equations can then be refined by expanding the velocity distribution to higher order in Kn (Agarwal *et al.* 2001). However, for granular materials, this "Burnett" expansion must also account for non-trivial effects of particle inelasticity (Sela & Goldhirsch 1998; Kumaran 2006).

Mitrano *et al.* (2013) suggest that such laborious approach might be avoided, while preserving clusters as a natural instability of the hydrodynamic equations. To show this, they juxtapose results from MD simulations similar to Goldhirsch & Zanetti (1993) and a numerical integration of the hydrodynamic equations that Garzó & Dufty (1999) derived for small Kn (figure 1). The linear stability analysis of Garzó (2005) yields a scale that Mitrano *et al.* (2012) had interpreted as a cluster length. As Brilliantov *et al.* (2004) suggested, clusters materialize in systems large enough to contain them.

In short, continuum equations at the Navier-Stokes order appear to initiate realistic granular clusters. This surprises Mitrano *et al.* (2013) for two reasons. First, vortices observed at the onset of clusters produce velocity correlations contradicting the molecular chaos assumption. Then, clusters comprise regions of large Kn that stretch equations beyond limits of their derivation.

3. Future

The article of Mitrano *et al.* (2013) fits within a fertile line of observations and models on inhomogeneities in granular media. Their remarks suggest that complicated extensions of hydrodynamic equations to finite Knudsen number may not be necessary to capture cluster onset. This is reminiscent of a debate in gas dynamics, whereby adjustments to the Navier-Stokes equations (Brenner 2005) could capture features of shock waves (Holian *et al.* 1993; Greenshields & Reese 2007), thus avoiding Burnett-order considerations despite recorded failures of the Navier-Stokes equations (Alsmeyer 1976). This simplification will be welcomed by those attempting to unify theories of clusters arising from fluid-particle interactions and granular collisions away from walls.

REFERENCES

Agarwal, R. K., Yun, K.-Y. & Balakrishnan, R. 2001 Beyond Navier-Stokes: Burnett equations for flows in the continuum-transition regime. *Phys. Fluids* **13**, 3061–3085.

Agrawal, K., Loezos, P. N., Syamlal, M. & Sundaresan, S. 2001 The role of meso-scale structures in rapid gas-solid flows. *J. Fluid Mech.* **445**, 151–185.

Alsmeyer, H. 1976 Density profiles in argon and nitrogen shock waves measured by the absorption of an electron beam. *J. Fluid Mech.* **74**, 497–613.

Bec, J., Biferale, L., Cencini, M., Lanotte, A. S. & Toschi, F. 2010 Intermittency in the velocity distribution of heavy particles in turbulence. *J. Fluid Mech.* **646**, 527–536.

Brenner, H. 2005 Navier-Stokes revisited. *Physica A* **349**, 60–132.

Brilliantov, N., Salueña, C., Schwager, T & Pöschel, T. 2004 Transient structures in a granular gas. *Phys. Rev. Lett.* **93**, 134301.

Capecelatro, J. & Desjardins, O. 2013 An Euler-Lagrange strategy for simulating particle-laden flows. *J. Comput. Phys.* **238**, 1–31.

Chun, J., Koch, D. L., Rani, S. L., Ahluwalia, A. & Collins, L. R. 2005 Clustering of aerosol particles in isotropic turbulence. *J. Fluid Mech.* **536**, 219–251.

Conway, S. L. & Glasser, B. J. 2004 Density waves and coherent structures in granular Couette flows. *Phys. Fluids* **16**, 509–529.

Desjardins, O., Fox, R. O. & Villedieu, P. 2008 A quadrature-based moment method for dilute fluid-particle flows. *J. Comput. Phys.* **227**, 2514–2539.

Doronzo, D. M., Martí, J., Sulpizio, R. & Dellino, P. 2012 Aerodynamics of stratovolcanoes during multiphase processes. *J. Geophys. Res.* **117**, B01207.

Duncan, K., Mehlig, B., Östlund, S. & Wilkinson, M. 2005 Clustering by mixing flows. *Phys. Rev. Lett.* **95**, 240602.

Fox, R. O. 2012 Large-Eddy-Simulation tools for multiphase flows. *Annu. Rev. Fluid Mech.* **44**, 47–76.

Garzó, V. 2005 Instabilities in a free granular fluid described by the Enskog equation. *Phys. Rev. E* **72**, 021106.

Garzó, V. & Dufty, J. W. 1999 Dense fluid transport for inelastic hard spheres. *Phys. Rev. E* **59**, 5895–5911.

Goldhirsch, I. & Zanetti, G. 1993 Clustering instability in dissipative gases. *Phys. Rev. Lett.* **70**, 1619–1622.

Greenshields, C. J. & Reese, J. M. 2007 The structure of shock waves as a test of Brenner's modifications to the Navier-Stokes equations. *J. Fluid Mech.* **580**, 407–429.

Helland, E., Bournot, H., Occelli, R. & Tadrist, L. 2007 Drag reduction and cluster formation in a circulating fluidised bed. *Chem. Eng. Sci.* **62**, 148–158.

van der Hoef, M. A., van Sint Annaland, M., Deen, N. G. & Kuipers, J. A. M. 2008 Numerical simulation of dense gas-solid fluidized beds: A multiscale modeling strategy. *Annu. Rev. Fluid Mech.* **40**, 47–70.

Holian, B. L., Patterson, C. W., Mareschal, M. & Salomons, E. 1993 Modeling shock waves in an ideal gas: Going beyond the Navier-Stokes level. *Phys. Rev. E* **47**, R24–R27.

Hopkins, M. A., Jenkins, J. T. & Louge, M. Y. 1993 On the structure of three-dimensional shear flows. *Mech. Mat.* **16**, 179–187.

Jenkins, J. T. & Richman, M. W. 1988 Plane simple shear of smooth inelastic circular disks: the anisotropy of the second moment in the dilute and dense limits. *J. Fluid Mech.* **192**, 313–328.

Jenkins, J. T. & Zhang, C. 2002 Kinetic theory for identical, frictional, nearly elastic spheres. *Phys. Fluids* **14**, 1228–1235.

Kumaran, V. 2006 The constitutive relation for the granular flow of rough particles, and its application to the flow down an inclined plane. *J. Fluid Mech.* **561**, 1–42.

Longmire, E. K. & Eaton, J. K. 1992 Structure of a particle-laden round jet. *J. Fluid Mech.* **236**, 217–257.

Lun, C. K. K. 1991 Kinetic theory for granular flow of dense, slightly inelastic, slightly rough spheres. *J. Fluid Mech.* **233**, 539–559.

Maxey, M. R. 1987 The gravitional settling of aerosol particles in homogeneous turbulence and random flow fields. *J. Fluid Mech.* **174**, 441–465.

McNamara, S. 1993 Hydrodynamic modes of a uniform granular medium. *Phys. Fluids A* **5**, 3056–3070.

Mitrano, P. P., Garzó, V., Hilger, A. M., Ewasko, C. J. & Hrenya, C. M. 2012 Assessing a hydrodynamic description for instabilities in highly dissipative, freely cooling granular gases. *Phys. Rev E* **85**, 041303.

Mitrano, P. P., Zenk, J., Benyahia, S., Galvin, J., Dahl, S. & Hrenya, C. 2013 Kinetic-theory predictions of clustering instabilities in granular flows: beyond the small Knudsen regime. *J. Fluid Mech.* **xx**, xx–xx.

Pan, L. & Padoan, P. 2010 Relative velocity of inertial particles in turbulent flows. *J. Fluid Mech.* **661**, 73–107.

Pepiot, P. & Desjardins, O. 2012 Numerical analysis of the dynamics of two- and three-dimensional fluidized bed reactors using an Euler-Lagrange approach. *Powder Tech.* **220**, 104–121.

Sela, N. & Goldhirsch, I. 1998 Hydrodynamic equations for rapid flows of smooth inelastic spheres, to Burnett order. *J. Fluid Mech.* **361**, 41–74.

Simonin, O., Zaichik, L. I., Alipchenkov, V. M. & Février, P. 2006 Connection between two statistical approaches for the modelling of particle velocity and concentration distributions in turbulent flow: The mesoscopic Eulerian formalism and the two-point probability density function method. *Phys. Fluids* **18**, 125107.

Wylie, J. J. & Koch, D. L. 2000 Particle clustering due to hydrodynamic interactions. *Phys. Fluids* **12**, 964–970.

Wylie, J. J., Koch, D. L. & Ladd, A. J. C. 2003 Rheology of suspensions with high particle inertia and moderate fluid inertia. *J. Fluid Mech.* **480**, 95–118.

Xu, H., Louge, M. & Reeves, A. 2003 Solutions of the kinetic theory for bounded collisional granular flows. *Continuum Mech. Thermodyn.* **15**, 321–349.

Xu, H., Verberg, R., Koch, D. L. & Louge, M. Y. 2009 Dense, bounded shear flows of agitated solid spheres in a gas at intermediate Stokes and finite Reynolds numbers. *J. Fluid Mech.* **618**, 181–208.