The Measurement of Particle Concentration with Optical Fiber Probes

by

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SUMMARY

Optical fibers are frequently used to measure particle concentration in dense gas-solid flows, because they are simple, they yield strong signals, and they create minimum disturbances, see for example, Hartge, Rensner, and Werther [*Chem. Ing. Tech.* **61**, 744-745 (1989)]. Despite their apparent simplicity, their signals do not always yield an unequivocal measurement of particle concentration.

Using a Monte-Carlo technique, we simulate the optical system consisting of the fiber and the suspension. We predict the response of the instrument at several volume fractions and the extent of its measurement volume. We examine in detail the performance of a single step index fiber and a design aimed at limiting the size of the measurement volume, which includes two converging emission and detection fibers. We find that probes with distinct emission and detection regions exhibit blind spots that may produce ambiguous signals, particularly for small particles. We reformulate the simple analysis of Rensner and Werther [*Part. Part. Syst. Charact.* **10**, 48-55 (1993)] in an integral form to interpret our results and to explain the behavior of these probes.

INTRODUCTION

Time-dependent measurements of particle volume fraction are essential to understand the flow of dense suspensions in fluidized beds, sedimentation, pneumatic and slurry transport lines. For suspensions with particle volume fraction greater than a few percent, the optical depth is of the order of several particle diameters. As a result, line-of-sight techniques are impractical. Instead, local measurements of particle volume fraction in dense suspensions are often achieved using optical fiber sensors.

Optical fibers are simple, they yield high signal-to-noise ratios, and they create minimum disturbances in the flow. However, they require careful calibration. Using Monte-Carlo simulations, Lischer and Louge (1992) showed that the output and accuracy of single fiber sensors in a suspension of spheres increase with decreasing sphere diameter and with increasing NA of the fiber. Thus it is generally advantageous to employ optical fibers with a diameter much larger than that of the particles. They also found that the output increases when the ratio of the indices of refraction of the sphere and the suspending medium is increased. In this context, they observed that suspensions of transparent particles in a gas produce signals of considerably different character than similar suspensions in a liquid. Therefore, they warned that immersing optical fiber sensors in water may produce a misleading calibration for gas suspensions. Finally, because the measurement volume increases with average interparticle distance, they suggested that in dilute suspensions optical fiber sensors may be sensitive to the structure deep in the flow.

Rensner and Werther (1993) proposed a simple analysis to predict the behavior of single optical fiber probes in particulate suspensions. As the present paper will show, an integral version of this model captures the general trends observed in the present simulations.

We begin with a brief description of the Monte-Carlo algorithm that we use to test the behavior of various optical fiber arrangements. We then improve upon the formulation of Rensner and Werther and exploit the resulting integral model to understand and contrast the response of a single optical fiber and that of a probe with two distinct emission and detection fibers.

MONTE-CARLO SIMULATION

This simulation predicts the fraction of the monochromatic radiant energy emitted by one or several multimode, step index optical fibers returning to one or several fibers from an isotropic, homogeneous, random suspension of smooth, monodisperse, spherical particles. Its original incarnation is described in detail by Lischer and Louge (1992). For the present study, several improvements have been implemented. The first permits to simulate arrangements involving several emission and detection fibers with arbitrary orientations. This is particularly useful to examine the behavior of systems like those of Reh and Li (1990), which consist of separate converging emission and detection fibers.

The second adds other optical components to the system such as windows and lenses. Finally, like the earlier simulations of Lischer and Louge, the new simulations model the suspension as an ensemble of cubes each of which contains a single sphere. The improvement is that the domain is now periodic, so that photons that leave through a boundary of the domain reappear in its next periodic image. For suspensions where the particle size is small compared with the fiber dimension, this method requires less computational memory.

SIMPLE OPTICAL MODEL

The interpretation of the simulation results is facilitated by a simple model recently proposed by Rensner and Werther (1993) for a probe consisting of a single optical fiber. We begin this section with a simplified description of the model, which we reformulate in an integral form. Because integrals are easier to manipulate than the series expansions of Rensner and Werther, the new formulation makes the model far more instructive.

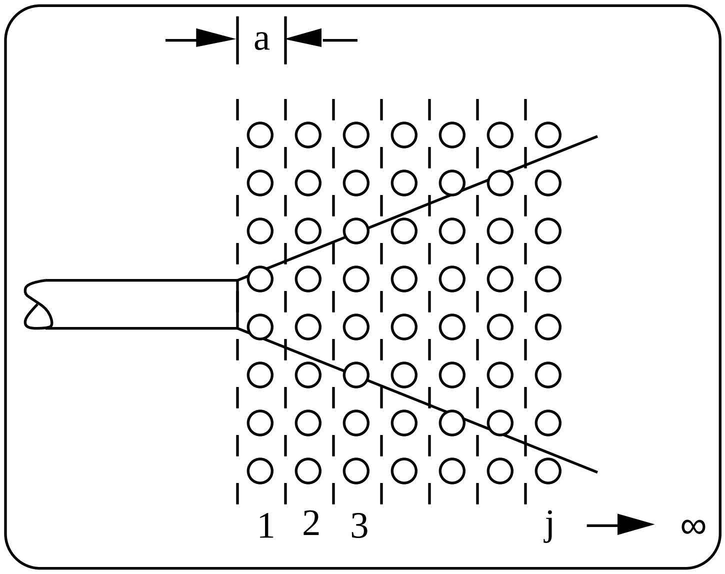


Fig. 1. The model of Rensner and Werther (1993).

Rensner and Werther assume that the suspension of volume fraction ν is composed of spheres of diameter d arranged in a cubic lattice of wavelength

$$a = d(\pi/6\nu)^{1/3}. \quad (1)$$

The optical axis of the fiber is aligned with one of the principal direction of the lattice (Fig. 1). Each layer extinguishes a fraction of the light proportional to the relative cross-section f that it presents to planar waves emitted by the fiber,

$$f = (\pi/4)\,(d/a)^3 = (\pi/4)\,(6\nu/\pi)^{2/3}. \quad (2)$$

In this ideal formulation of the extinction, the suspension behaves as a homogeneous, isotropic, absorbing medium. The fraction of light extinguished per unit length is

$$\kappa\, f/a = (3/2)\,\kappa\,(\nu/d), \quad (3)$$

where κ is a constant of proportionality typically of o(2). In this ideal medium, the initial intensity i_0 is reduced to the value i at the distance x according to Beer's law,

$$i/i_0 = \exp[-\,(3/2)\,\kappa\,\nu\,x/d]\,. \quad (4)$$

Rensner and Werther assume that backscattering from each layer is proportional to the relative fraction f, so the amount of backscattering per unit length of the suspension is

$$\lambda f/a = (3/2)\,\lambda\,(\nu/d), \quad (5)$$

where λ is a constant < 1. They add the contribution of each successive layer in the suspension to calculate the total amount of light returned to the optical fiber as an infinite series. They recognize that a layer at a distance x from the probe is illuminated with only a fraction of the incident light, and that the resulting backscattered light is further extinguished through the suspension upon its return to the fiber. Thus the contribution of an infinitesimal layer of width dx at the distance x is $\exp[-\frac{3}{2}\kappa\frac{\nu}{d}2x]\,\frac{3}{2}\lambda\frac{\nu}{d}\,dx$.

To capture the divergence of light from the optical fiber, Rensner and Werther introduce a form factor equal to the fraction of the energy returned to the fiber from an infinite screen normal to the optical axis and located at a distance x from the fiber. The idea is to regard the backscattering layer as a screen and the suspension as a homogeneous, isotropic, dielectric, absorbing medium. The form factor $\varphi(x/D)$ is a function of the NA of the fiber and the ratio x/D, where D is the fiber diameter.

In this formulation, the total amount of light returned to the fiber is the integral over all elementary layers that contribute to the backscattering,

$$\frac{i}{i_0} = \int_0^\infty \varphi[\frac{x}{D}]\,\exp[-\frac{3}{2}\kappa\frac{\nu}{d}2x]\,\frac{3}{2}\lambda\frac{\nu}{d}\,dx\,. \tag{6}$$

It is convenient to rewrite the above and highlight dimensionless groups,

$$\frac{i}{i_0} = \int_0^\infty \varphi[x^\dagger]\,\exp[-2\kappa\,\nu^\dagger\,x^\dagger]\,\lambda\nu^\dagger\,dx^\dagger\,, \tag{7}$$

where $\nu^\dagger \equiv (3/2)\,\nu\,(D/d)$ and $x^\dagger \equiv (x/D)$. A natural length scale arises from the exponential decay of the light transmitted,

$$L^\dagger \equiv (1/2\kappa\nu^\dagger) = d/3\kappa\nu D. \tag{8}$$

To highlight the role of the form factor φ, it is instructive to imagine that φ is everywhere unity. For an optical fiber system this is clearly impossible, as it would correspond to a planar wave traveling in and out of the suspension. In this case, (7) would integrate to $i/i_0 = \lambda/2\kappa$, which is independent of volume fraction and particle size. Thus a vigorous dependence of the form factor upon distance is essential for the success of the optical probe through a significant dependence of i/i_0 on ν.

At low volume fractions, the exponential in (7) is nearly unity. Thus, as $\nu^\dagger \to 0$, the fraction of light returned grows linearly with particle volume fraction,

$$i/i_0 \sim \lambda\, \nu^\dagger \int_0^\infty \varphi[x^\dagger]\, dx^\dagger .$$

At large volume fraction, it is convenient to integrate (7) by parts,

$$i/i_0 = \frac{\lambda}{2\kappa}[\varphi_0 + \int_0^\infty \frac{\partial\varphi}{\partial x^\dagger} \exp[- 2\kappa\, \nu^\dagger\, x^\dagger]\, dx^\dagger] \,, \tag{9}$$

where φ_0 is the value of the function $\varphi[x/D]$ at the tip of the fiber, x=0. Thus the derivative of (i/i_0) with respect to $\nu^\dagger$ is

$$\frac{\partial(i/i_0)}{\partial\nu^\dagger} = \lambda \int_0^\infty (- \frac{\partial\varphi}{\partial x^\dagger})\, x^\dagger \exp[- x^\dagger/L^\dagger]\, dx^\dagger \,, \tag{10}$$

and its second derivative is

$$\frac{\partial^2(i/i_0)}{\partial\nu^{\dagger 2}} = \lambda \int_0^\infty (\frac{\partial\varphi}{\partial x^\dagger})\, 2\kappa x^{\dagger 2} \exp[- x^\dagger/L^\dagger]\, dx^\dagger . \tag{11}$$

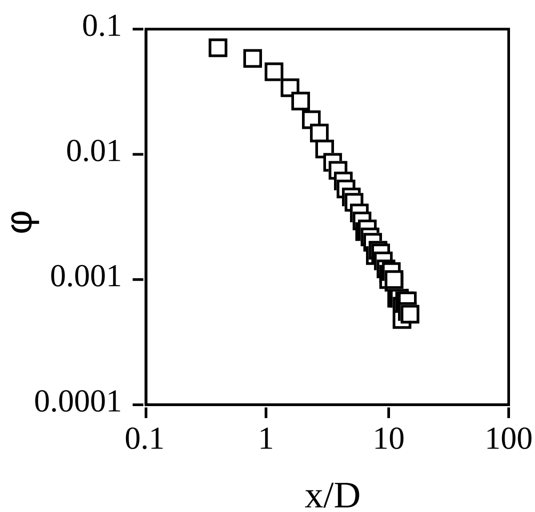


Fig. 2. Form factor for a single step-index optical fiber of 0.4 NA.

For single fibers, the form factor φ decreases monotonically with x for all $x \geq 0$ (Fig. 2). In this case, as predicted by (10), i/i_0 grows with increasing volume fraction and decreasing sphere diameter. Further, Eq. (11) shows that the slope of i/i_0 versus $\nu^\dagger$ rolls off at increasing values of the volume fraction. Thus the simple model of Eq. (7) correctly captures effects commonly observed with single fibers (Fig. 3).

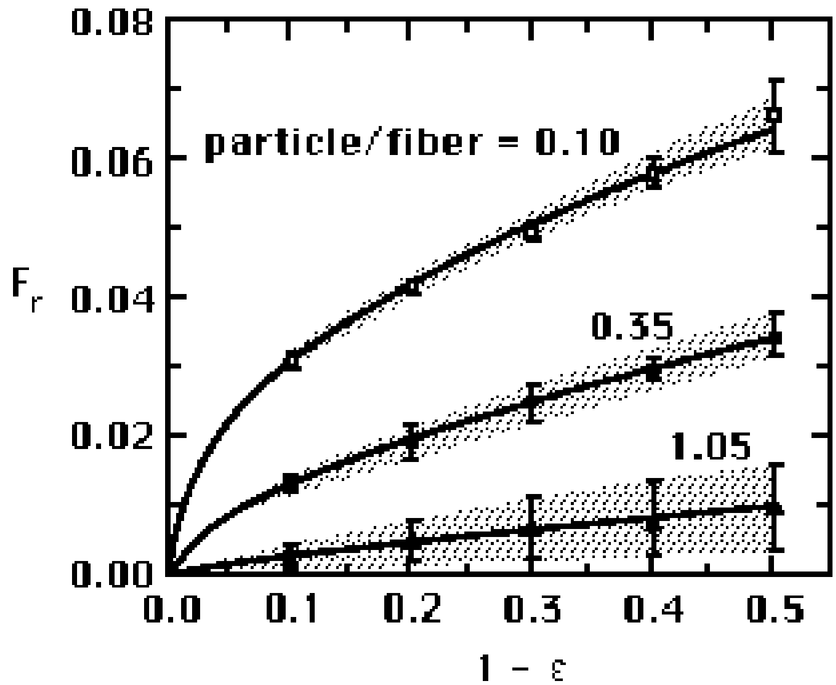
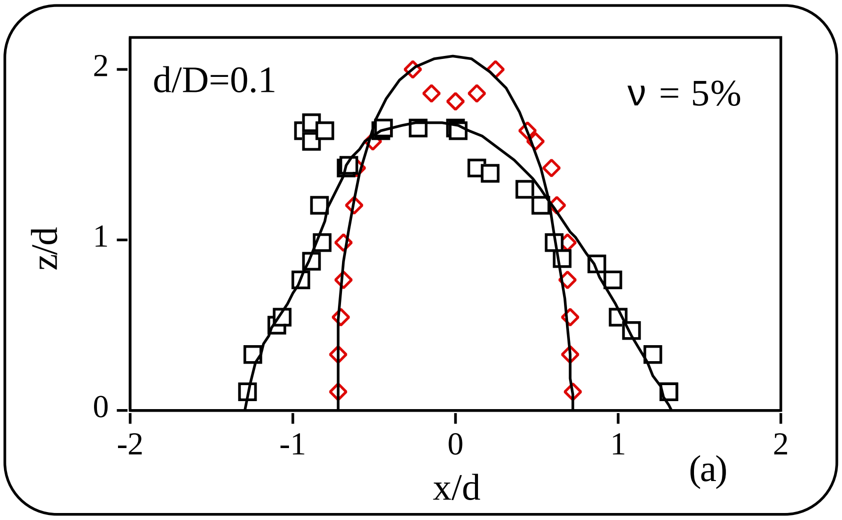


Fig. 3. Results of simulation for d/D = 1.05 (triangles), 0.35 (circles), and 0.10 (squares) for suspensions of glass spheres in air and a fiber of 0.37 NA. Error bars represent the sample standard deviation of the fraction returning $F_r = i/i_0$ for 10 randomly simulated particle placements. The solids lines are least-squares fits of the form $F_r = k\,(1-\varepsilon)^m$, where (1-ε) is the solid volume fraction [from Lischer and Louge, 1992)].

THE ROLE OF "BLIND SPOTS"

The principal difficulty with single optical fiber probes is that their measurement volume becomes excessive at low volume fractions. As a result, their calibrations against quantitative instruments like capacitance probes (Lischer and Louge, 1992) may depend upon the structure deep in the flow, which is generally affected by overall flow conditions.

To remedy this problem, Reh and Li (1990) proposed a converging arrangement of separate emission and detection fibers. As our simulations indicate (Fig. 4), the new arrangement yields a more confined measurement volume near the fiber. However, under particular circumstances, the light returned to the detection fiber may decrease with increasing volume fraction (Fig. 5). This behavior clearly renders the instrument ambiguous, as two different volume fractions may produce the same output signal.



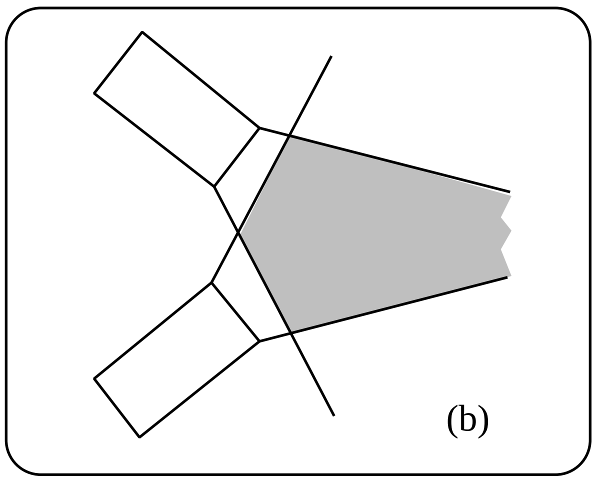


Fig. 4. (a) Cross-section of the measurement volumes of a single fiber (diamonds) and converging emission and detection fibers (squares) along the plane of symmetry of the system; (b) relative dimensions of the converging probe.

Note however that Reh and Li did not use the probe with aspect ratios sketched in Fig. (4b). Our simulations indicate that their specific design does not exhibit these problems, at least in the range of particle sizes that they employed. Nevertheless, the reader will appreciate from the following discussions how potentially ambiguous the converging design may be.

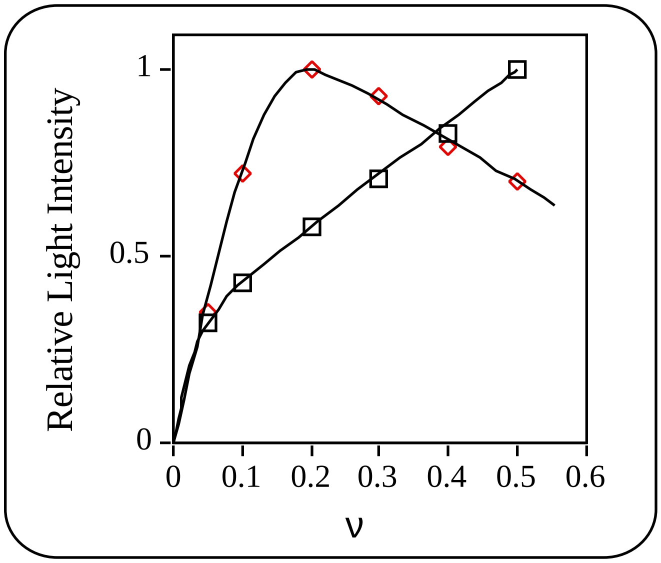


Fig. 5. Simulated fraction of light returning *vs* volume fraction for the single fiber (squares) and the converging arrangement (diamonds) of Fig. 4. For comparison's sake the values of i/i_0 for both cases are divided by their maximum value.

To understand the form of the output from the two converging fibers of Fig. 4, we return to our integral version of the model of Rensner and Werther and compare the behavior of two simpler test cases. The first is a single step index fiber of 0.4 NA. The second is composed of two parallel, identical fibers, the axes of which are separated by a distance s equal to their diameter. In the model, the principal difference between the two cases is that the form factor of the twin fibers vanishes at the fibers' face and possesses a maximum at $x^{\dagger}=\alpha$ (Fig. 6).

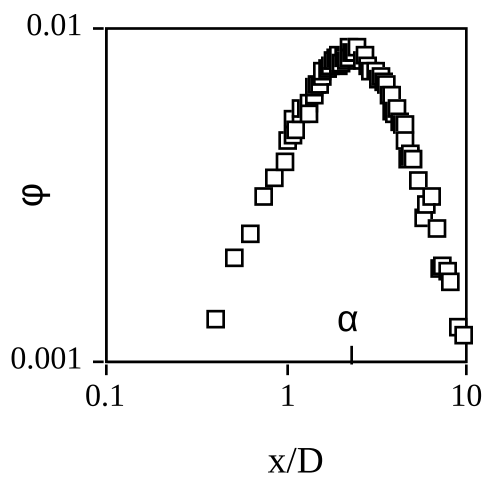


Fig. 6. Form factor for twin step-index fibers of 0.4 NA separated by a distance s/D=1.

Two limits may thus arise with the twin fibers. In the first, the characteristic length of transmission is small compared with α. This corresponds to small particles. Here, the exponential in (10) quickly vanishes while the derivative of the form factor with $x^\dagger$ is positive. Thus for $L^\dagger \ll \alpha$, $\partial(i/i_0)/\partial\nu^\dagger < 0$ and the probe signal decreases with increasing particle volume fraction. The probe therefore exhibits an ambiguous response.

In contrast, for $L^\dagger \gg \alpha$, the exponential has barely decayed long after the form factor has reached its maximum. Thus with $\exp[-x^\dagger/L^\dagger] \sim 1$, Eq.(10) is approximately

$$\frac{\partial(i/i_0)}{\partial\nu^\dagger} \sim -\lambda \int_0^\infty x^\dagger d\varphi = -\lambda\left[x^\dagger\varphi\right]_0^\infty + \lambda \int_0^\infty \varphi dx^\dagger . \tag{12}$$

Because φ generally decays faster than at least $x^{\dagger 2}$, the first term vanishes and in this limit

$$\frac{\partial(i/i_0)}{\partial\nu^\dagger} \sim \lambda \int_0^\infty \varphi dx^\dagger > 0. \tag{13}$$

Thus for large enough spheres the signal from the twin probes may yet increase with particle volume fraction.

In general, optical probes with distinct emission and detection fibers are prone to ambiguous signals as $\partial(i/i_0)/\partial\nu^\dagger$ may become negative at high ν. This behavior results from the two competing effects of transmission and backscattering. As particle volume fraction grows, transmission decreases while backscattering increases. The form factor thus determines which of the two effect prevails. If the optical fiber is unique, the backscattering dominates even at large volume fraction. There, transmission through the suspension is minimum, as only the first few layers contribute to the output signal through backscattering.

In the case of distinct emission and detection fibers, there are “blind spots” in the field of view of the detection fiber i.e., light backscattered in front of the emission fiber may not be detected without traveling first through the suspension. In this case transmission prevails over backscattering. Thus at large volume fractions or for small particles, the probe response follows that of transmission, which decreases with ν. Nevertheless, if the spheres are large enough, their first layers may still be located away from the blind spots near the probe. In this case, the signal may still exhibit growth with ν. For a practical system however, increasing the particle size to avoid an ambiguous response may be counterproductive, as signal uncertainty grows with particle size (Lischer and Louge, 1992).

CONCLUSIONS

In this paper, we have employed Monte-Carlo simulations to predict the behavior of various optical fiber systems for recording particle volume fraction. We have found that systems consisting of distinct emission and detection fibers may produce ambiguous signals that fail to increase monotonically with particle volume fraction. By reformulating the simple model of Rensner and Werther (1993) in an integral form, we have shown that these ambiguous signals are the result of “blind spots” in the field of view of the detection fibers. Because the effects of these blind spots depend upon particle size, we warn that optical probes with distinct emission and detection fibers should be carefully designed and calibrated for each suspension under study.

ACKNOWLEDGMENTS

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