Dense granular flows down inclines

Michel Louge

Cornell University

UNIVERSITE DE RENNES 1

Qatar National Research Fund

Member of Qatar Foundation

2nd IMA Conference on Dense Granular Flows

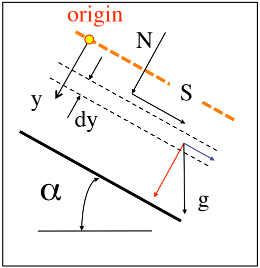
Isaac Newton Institute for Mathematical Sciences, Cambridge UK

July 1, 2013

Dense inclined granular flow

Steady, fully-developed force balance

Steady $\left(\partial / \partial t \equiv 0\right)$, fully-developed $\left(\partial / \partial x \equiv 0\right)$ flow



Force balance

$$\frac{dS}{dy} = \rho g \sin\alpha$$

$$\frac{dN}{dy} = \rho g \cos\alpha$$

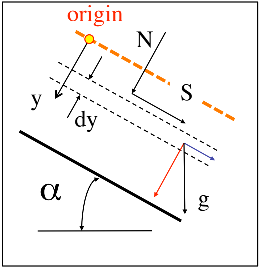
$$S = \rho g y \sin\alpha$$

$$N = \rho g y \cos\alpha$$

$$\mu_{\mathrm{eff}} = \frac{S}{N} = \tan\alpha$$

Newtonian viscous fluid

laminar flow



Force balance

$$\frac{dS}{dy} = \rho g \sin\alpha$$

$$\frac{dN}{dy} = \rho g \cos\alpha$$

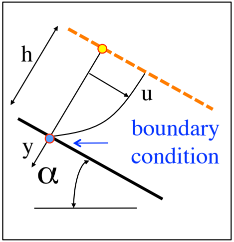
$$S = \rho g y \sin\alpha$$

$$N = \rho g y \cos\alpha$$

$$\mu_{\mathrm{eff}} = \frac{S}{N} = \tan\alpha$$

Constitutive relation

$$S = -\mu \frac{du}{dy}$$

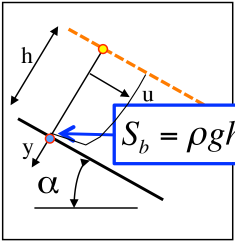


$$u = \frac{\rho g \sin\alpha}{2\mu}\left(h^2 - y^2\right)$$

$$\frac{\dot{m}}{W} = \left(\frac{\rho^2 g \sin\alpha}{3\mu}\right) h^3$$

Newtonian viscous fluid

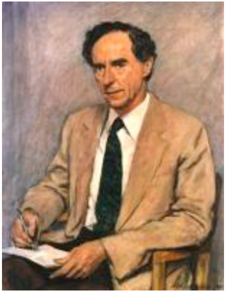
turbulent flow



$$S_b = \rho g h \sin\alpha = \text{cst} \times \rho \bar{u}^2$$

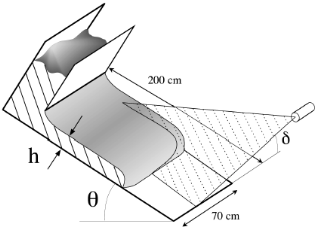
$$\text{cst} = \left(u^* / \bar{u}\right)^2 \approx 3\ 10^{-3}$$

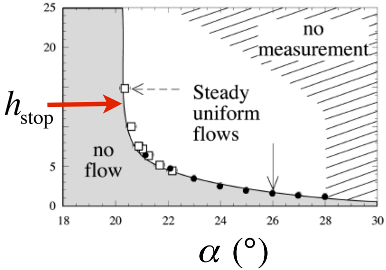
$$\frac{\dot{m}}{W} = \rho \bar{u} h = \rho \sqrt{\frac{g \sin\alpha}{\text{cst}}}\, h^{3/2}$$



George Batchelor

Shallow flows, far sidewalls, bumpy base

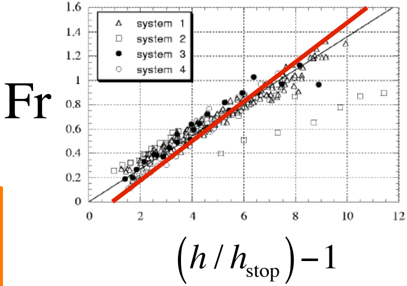




Pouliquen, Phys. Fluids 1999

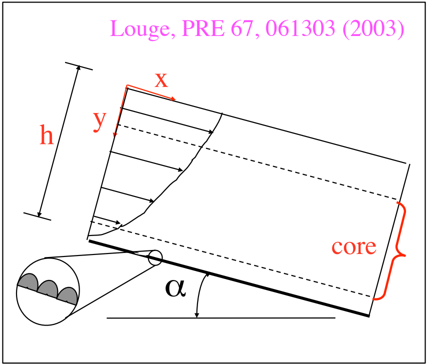
$$\frac{\dot{m}}{W} \propto \text{cst}\, \rho_s \bar{\nu} \frac{\sqrt{g\cos\alpha}}{h_{stop}} h^{5/2}$$

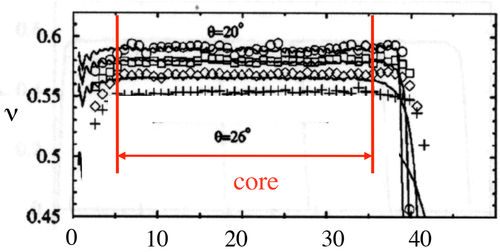
$$\text{Fr} \equiv \frac{\bar{u}}{\sqrt{N/\rho}} = \frac{\bar{u}}{\sqrt{gh\cos\alpha}}$$



Deboeuf, et al, PRL 2006

Shallow flows, far sidewalls, bumpy base





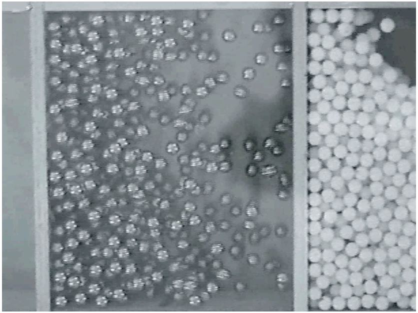
Silbert, et al. PRE 2001

$$\frac{\dot{m}}{W} \propto \text{cst}\ \rho_s \bar{\nu} \frac{\sqrt{g \cos\alpha}}{h_{\text{stop}}} h^{5/2}$$

$$20^\circ < \alpha < 26^\circ$$
$$0.595 > \nu > 0.545$$

Leo Silbert

Granular temperature



GRC
GLENN RESEARCH CENTER
at Lewis Field

"temperature" $T = \frac{1}{3}\,\overline{u'_i\, u'_i}$

fluctuation velocity u'_i

Transport coefficients

Stuart Savage

James Jenkins

viscosity $\mu = \rho_s d \sqrt{T} f_1(\nu; g_{12})$

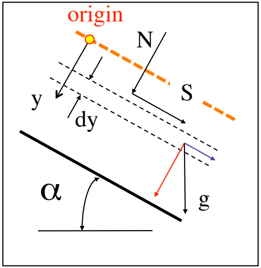
conductivity $\kappa = \rho_s d \sqrt{T} f_2(\nu; g_{12})$

dissipation $\gamma = f_3(\nu; g_{12}; e) \rho_s T^{3/2} / d$

equation of state $N = f_4(\nu; g_{12}) \rho_s T$

Kinetic theory cartoon

Ralph Bagnold



bumpy base

viscosity

$$S = \rho_s \nu g y \sin\alpha = -f_1 \rho_s d \sqrt{T}\,\frac{du}{dy}$$

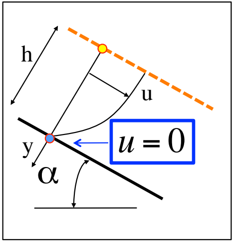
$f_1(\nu; g_{12})$

T = "granular temperature"

$f_4(\nu; g_{12})$

$$N = \rho_s \nu g y \cos\alpha = f_4 \rho_s T$$

equation of state



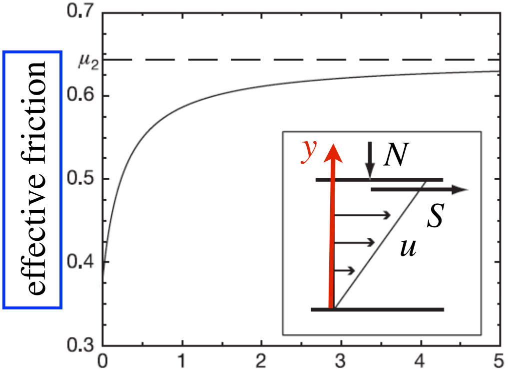
$$u = \frac{2}{3}\left(\frac{\sqrt{\nu f_4}}{f_1}\tan\alpha\right)\frac{\sqrt{g\cos\alpha}}{d}\left(h^{3/2} - y^{3/2}\right)$$

"Bagnold profile"

I

$$\frac{\dot{m}}{W} = \frac{2}{5}\,\frac{\nu^{3/2} f_4^{1/2}}{f_1}\,\frac{\rho_s g^{1/2}}{d}\,\frac{\sin\alpha}{\sqrt{\cos\alpha}}\,h^{5/2}$$

Inertial number



$$I \equiv \frac{|\mathrm{d}u/\mathrm{d}y|\; d}{\sqrt{N/\rho_s \nu}}$$

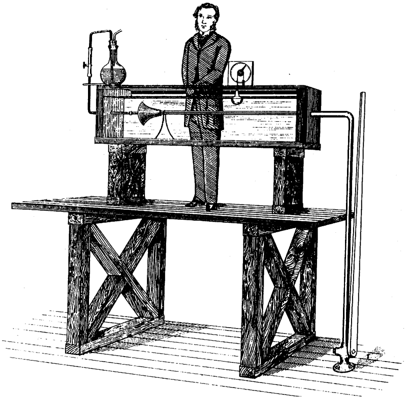
da Cruz, et al PRE 2005

Jop, Forterre & Pouliquen Nature 2006

GDR MiDi, Eur. Phys. J. E 2003

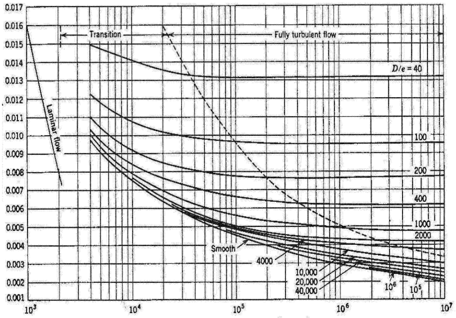
Chevoir, et al Powder Tech. 2009

Problem solved?



Osborne Reynolds, 1883

friction coefficient

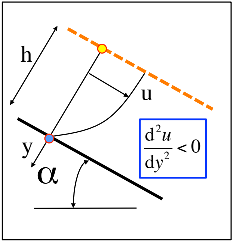


$$\mathrm{Re} \equiv \frac{ud}{\mu/\rho}$$

Profile concavity and viscosity

$$S = \rho_s \nu g y \sin\alpha = -\mu \frac{du}{dy}$$

$$\frac{d^2u}{dy^2} = -\rho_s g \sin\alpha \left(\frac{\nu y}{\mu}\right)\left[\frac{\nu'}{\nu} + \frac{1}{y} - \frac{\mu'}{\mu}\right]$$



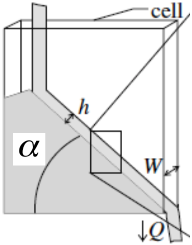
$$\frac{d^2u}{dy^2} < 0 \Leftrightarrow \frac{d\ln\mu}{d\ln y} < \frac{d\ln\nu}{d\ln y} + 1$$

bumpy base $\quad \frac{d\ln\mu}{d\ln y} = \frac{1}{2} \qquad \frac{d\ln\nu}{d\ln y} = 0$

Moderate increase in viscosity with depth for core flows over a bumpy base

Inverted concavity for a soft base

Sidewall-stabilized heap



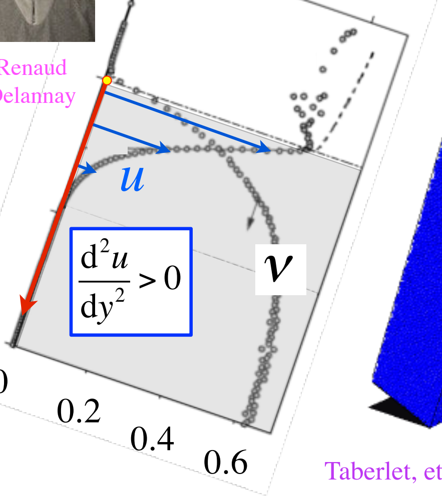
Renaud Delannay

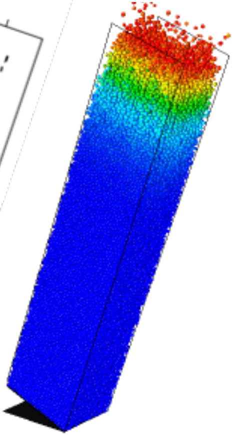
$$\frac{d^2u}{dy^2} > 0 \Leftrightarrow \frac{d\ln\mu}{d\ln y} > \frac{d\ln\nu}{d\ln y} + 1$$

Rapid increase in volume fraction and viscosity with depth.

Louge, et al. Powders & Grains 2005

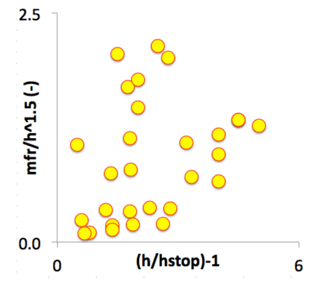
Richard, et al. PRL 2008



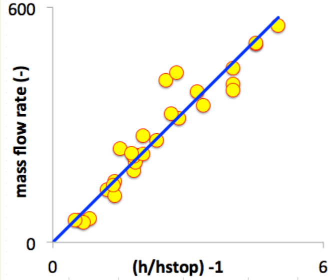


Taberlet, et al. PRL 2003

Role of the base



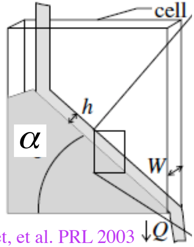
with inertial number scaling



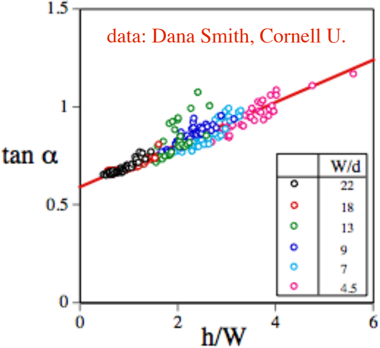
$$\frac{\dot{m}}{W} \approx \mathrm{cst} \times \rho_s d \sqrt{gd} \left(\frac{h}{h_{\mathrm{stop}}} - 1 \right)$$

Role of side walls

Sidewall-stabilized heap



Taberlet, et al. PRL 2003



$$\frac{\dot{m}}{W} \approx \text{cst} \times \rho_s W \sqrt{gW} \left(\frac{h}{W} - 1 \right)$$

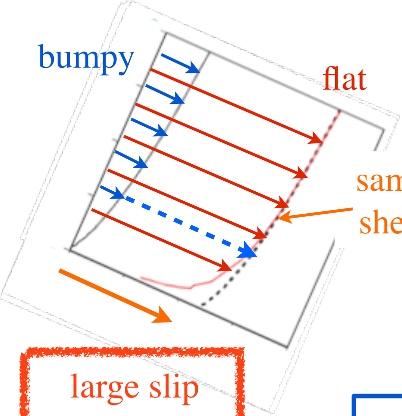
$$\tan\alpha = \mu_w \left(\frac{h}{W} \right) + \tan\alpha_{\min}$$

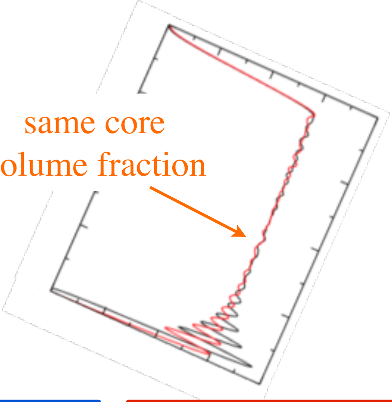
$$\frac{\dot{m}}{W} \approx \text{cst} \times \rho_s d \sqrt{gd} \left(\frac{h}{h_{\text{stop}}} - 1 \right)$$

soft base,
far side walls

Flat, frictional base

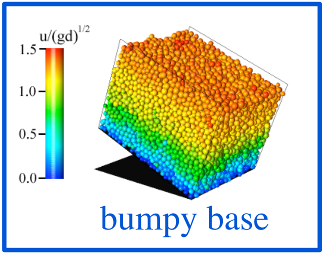
Delannay, et al, Nature Mat. 2007

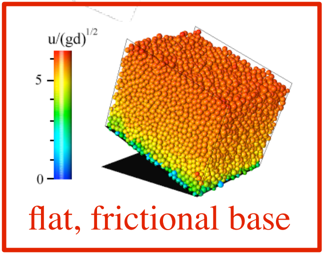




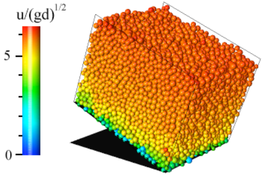
large slip
at the bottom

Simulations:
Nicolas Taberlet

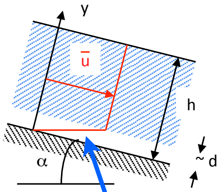




Flat, frictional base cartoon



Louge and Keast, Phys. Fluids 2001



$$T_0 \sim \frac{\bar{\nu}}{f_4(\nu_0)} g h \cos\alpha$$

$$S \sim \rho_s g h \bar{\nu} \sin\alpha \sim f_1(\nu_0)\rho_s d \sqrt{T_0}\,\frac{\bar{u}}{d}$$

$$\mathrm{Fr} = \frac{\bar{u}}{\sqrt{g h \cos\alpha}} \sim \left(\frac{\bar{\nu}^{1/2} f_4^{1/2}}{f_1}\right)\tan\alpha$$

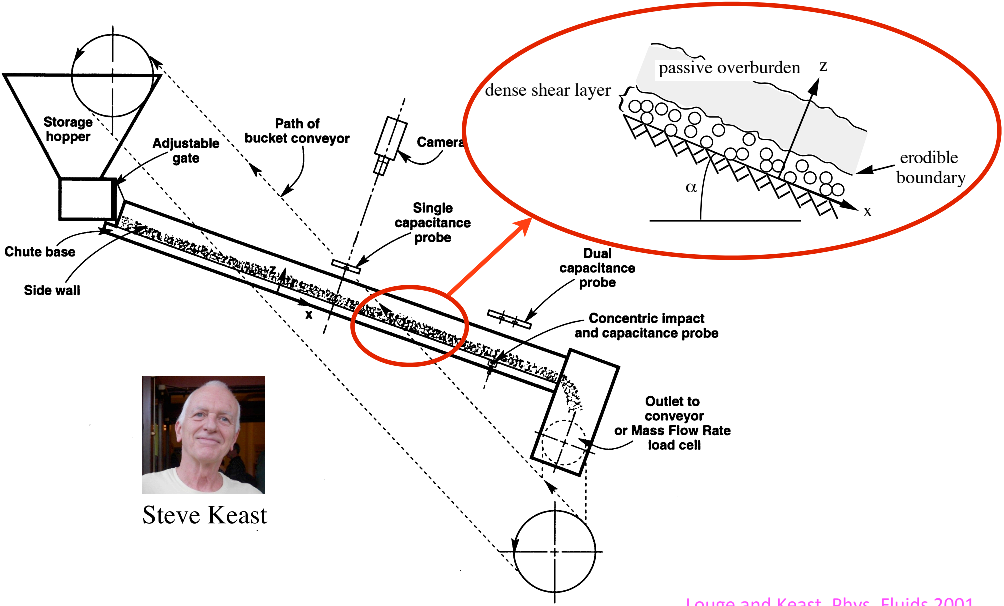
$$\frac{\dot{m}}{W} \sim \rho_s g^{1/2} \frac{\bar{\nu}^{3/2}\sin\alpha}{\sqrt{\cos\alpha}} \left(\frac{f_4(\nu_0)^{1/2}}{f_1(\nu_0)}\right) h^{3/2}$$

$T_0,\ \nu_0$

basal quantities

length scale

Flat, frictional base experiments



Steve Keast

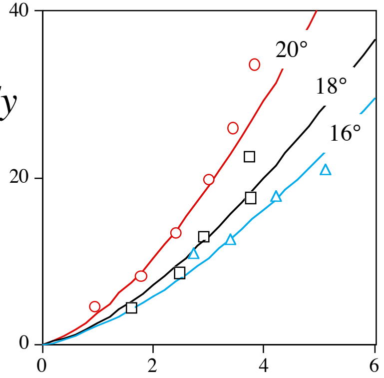
Louge and Keast, Phys. Fluids 2001

Flat, frictional base; shallow flows

Sustained flows exist at inclinations in the range 15.5° ≤ α ≤ 20°.

$$m^{\dagger} = \frac{1}{d\sqrt{gd}} \int_0^h u\nu \, dy$$

mass flowrate



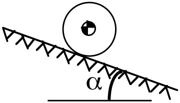
Roy Jackson

mass holdup $$H^{\dagger} = \frac{1}{d} \int_0^h \nu \, dy = \left(\frac{h}{d} \bar{\nu} \right)$$

Louge and Keast, Phys. Fluids 2001

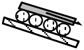
Steady flows require variable friction

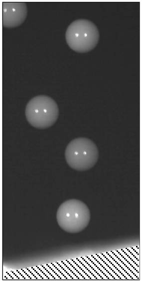
For steady, fully-developed flows: $S/N = \mu_{eff} = \tan\alpha$



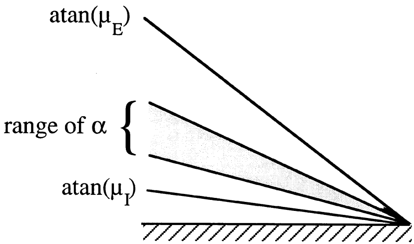
Forces are exerted at contact with the base:

$\mu_E = 0.59$





$\mu_I = 0.14$



$$S = S_I + S_E$$

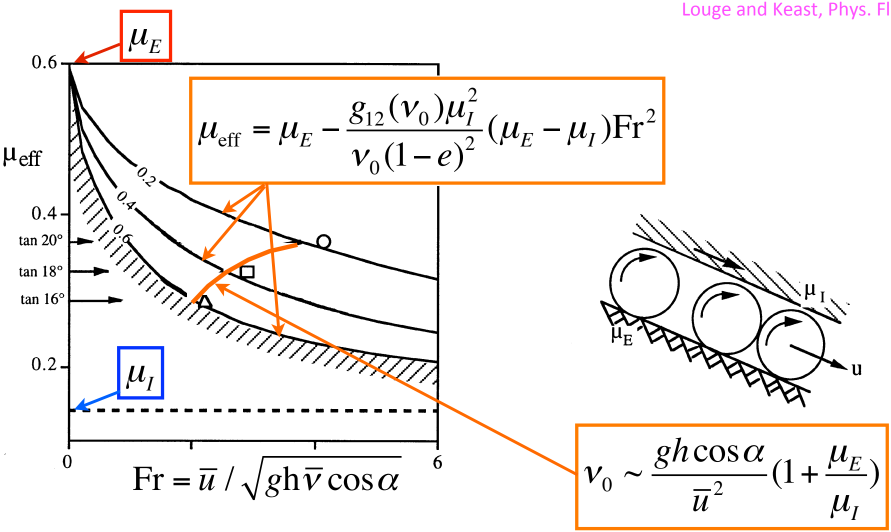
$$N = N_I + N_E$$

$$\mu_{eff} = S/N$$

Louge and Keast, Phys. Fluids 2001

Effective friction set by basal rolling

Louge and Keast, Phys. Fluids 2001



$$\mu_{\text{eff}} = \mu_E - \frac{g_{12}(\nu_0)\mu_I^2}{\nu_0(1-e)^2}(\mu_E - \mu_I)\text{Fr}^2$$

$$\nu_0 \sim \frac{gh\cos\alpha}{\bar{u}^2}\left(1 + \frac{\mu_E}{\mu_I}\right)$$

Fluctuation energy

$$0 = -\frac{dq}{dy} + S\frac{du}{dy} - \gamma$$

heat flux gradient

shear work rate

dissipation

$$q = -\kappa \frac{dT}{dy}$$

$$\kappa = f_2 \rho_s d \sqrt{T}$$

$$S = f_1 \rho_s d \sqrt{T} \frac{du}{dy}$$

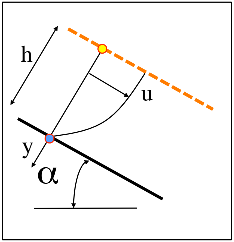
$$\gamma = f_3 \rho_s T^{3/2} / d$$

Flux conundrum

$$N \approx \rho_s \bar{v} g \cos\alpha y$$

$$S \approx \rho_s \bar{v} g \sin\alpha y$$

$$T \approx \bar{v} g \cos\alpha y / f_4$$



$$y^* \equiv \frac{y}{d}$$

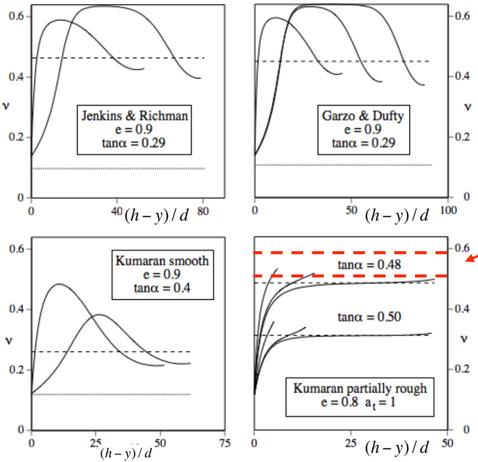
$$\frac{f_1}{f_4^{1/2}} \sqrt{y^*} \frac{d}{dy^*} \left\{ \frac{f_2}{f_4^{3/2}} \sqrt{y^*} \left[1 - \left(\frac{d \ln f_4}{d \ln \nu} \right) \left(\frac{d \ln \nu}{d \ln y^*} \right) \right] \right\} + y^{*2} \left[\tan^2\alpha - \frac{f_1 f_3}{f_4^2} \right] = 0$$

$$\left. \begin{aligned} &\frac{dq}{dy} \equiv 0 \\ &T \approx \bar{v} g \cos\alpha / f_4 \\ &q = -\kappa \frac{dT}{dy} \end{aligned} \right\} \frac{d\kappa}{dy} \equiv 0$$

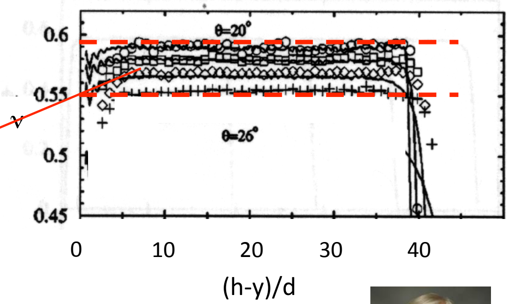
Invariant conductivity

$$0 = -\frac{dq}{dy} + S\frac{du}{dy} - \gamma$$

Consequence on the volume fraction



Silbert, et al. PRE 2001



Kumaran

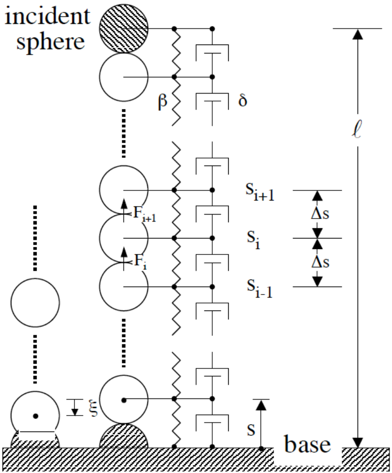
Jenkins & Richman 1985

Garzo & Dufty PRE 1999

Kumaran JFM 2006

Jim Dufty

Phonon conductivity



$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2} + D \frac{\partial^3 \xi}{\partial t \partial x^2}$$

$$c^2 = \frac{\beta d^2}{m} \qquad D = \frac{d^2 \delta}{m}$$

$$\tilde{\kappa} = \frac{3}{2} \rho_s \nu D \left[1 - \frac{j}{\sqrt{\varepsilon}} \right] \qquad \varepsilon \equiv \frac{D^2 \omega^2}{c^4}$$

$$\kappa \approx \sqrt{\frac{54}{\pi^3}} \nu \left(\frac{\ell}{d} \right) \sqrt{\rho_s \beta d} >> f_2 \rho_s d \sqrt{T}$$

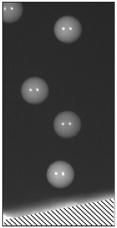
Louge, Gran. Mat. 2011

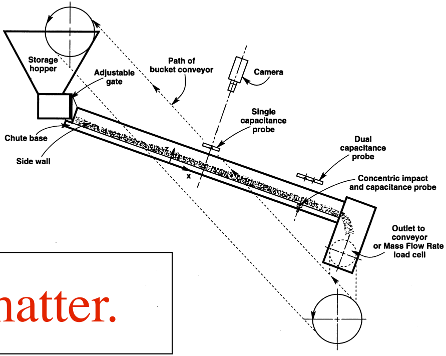
Mass flow rate versus flowing depth

flow type	n
soft, dissipative base	1
Newtonian fluid, turbulent	1.5
flat, frictional base	1.5
core over a bumpy base	2.5
Newtonian fluid, laminar	3

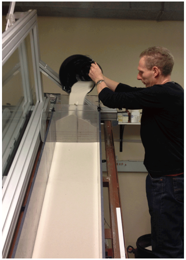
$$\frac{\dot{m}}{W} \propto H^{n}$$

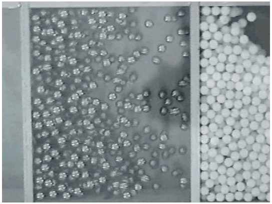
channel width W,
flowing depth H





Boundaries matter.





http://grainflowresearch.mae.cornell.edu