Granular flows down inclines:

how different bases produce widely different scaling of mass flow rate with depth

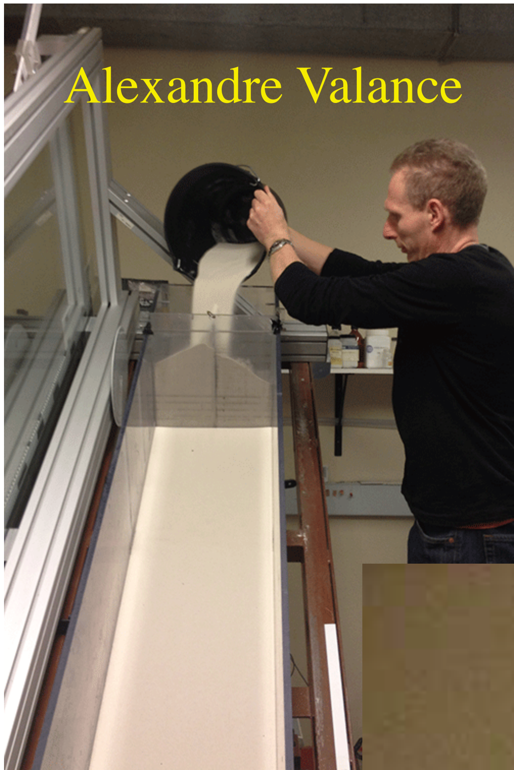
Michel Louge

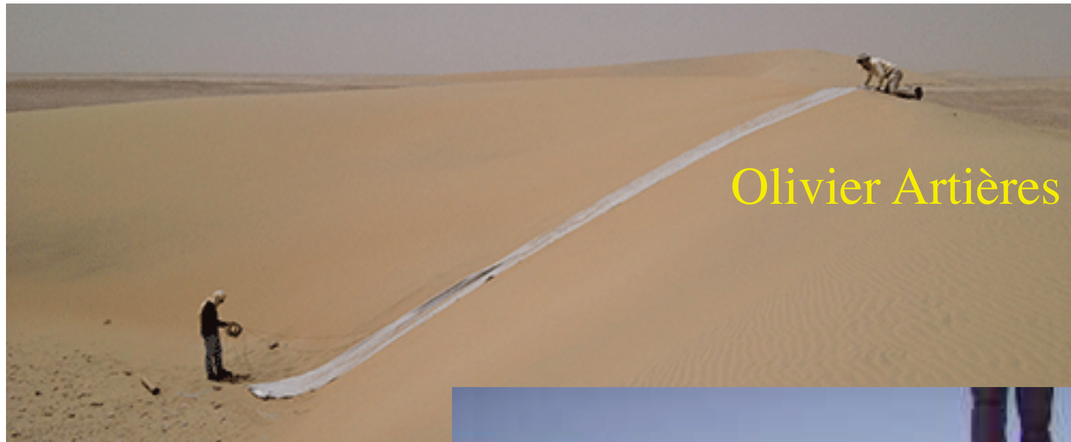
Second International Granular Flow Workshop, Guiyang, China

Tuesday, August 22, 2017

ML, A. Valance, P. Lancelot, R. Delannay, O. Artières, PRE 92, 022204 (2015)

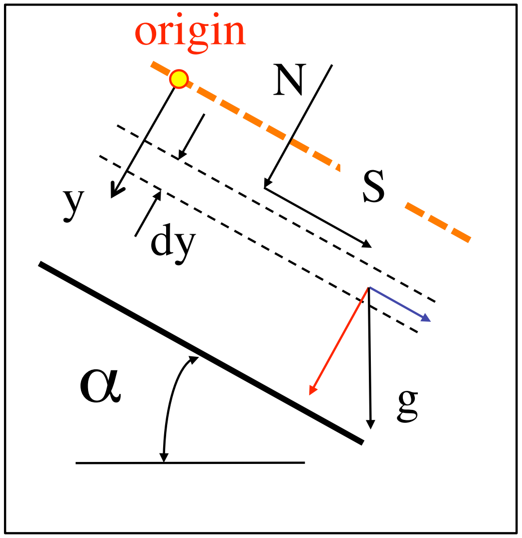
Dense inclined granular flow





Steady, fully-developed force balance

Steady $\left(\partial / \partial t \equiv 0\right)$, fully-developed $\left(\partial / \partial x \equiv 0\right)$ flow



Force balance

$$\frac{dS}{dy} = \rho g \sin \alpha$$

$$\frac{dN}{dy} = \rho g \cos \alpha$$

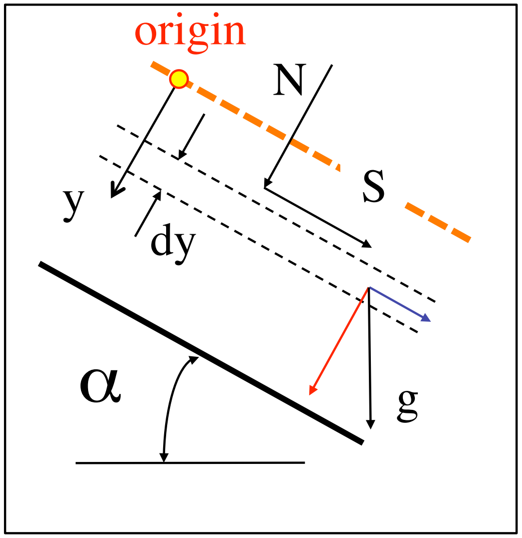
$$S = \rho g y \sin \alpha$$

$$N = \rho g y \cos \alpha$$

$$\mu_{\text{eff}} = \frac{S}{N} = \tan \alpha$$

Newtonian viscous fluid

laminar flow



Force balance

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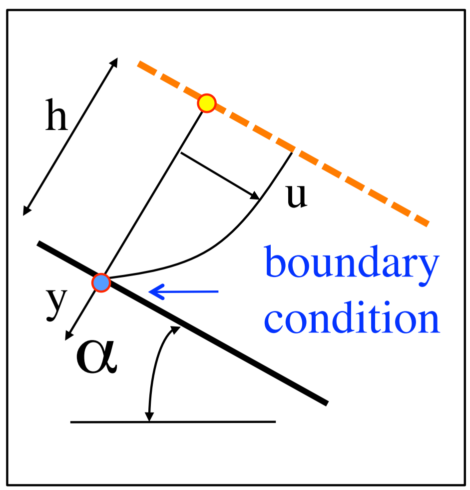
$$S = \rho g y \sin\alpha$$

$$N = \rho g y \cos\alpha$$

$$\mu_{\mathrm{eff}} = \frac{S}{N} = \tan\alpha$$

Constitutive relation

$$S = -\mu \frac{du}{dy}$$

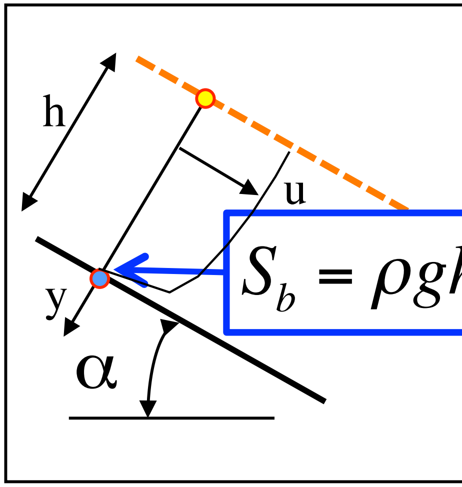


$$u = \frac{\rho g \sin\alpha}{2\mu}\left(h^2 - y^2\right)$$

$$\frac{\dot{m}}{W} = \left(\frac{\rho^2 g \sin\alpha}{3\mu}\right) h^3$$

Newtonian viscous fluid

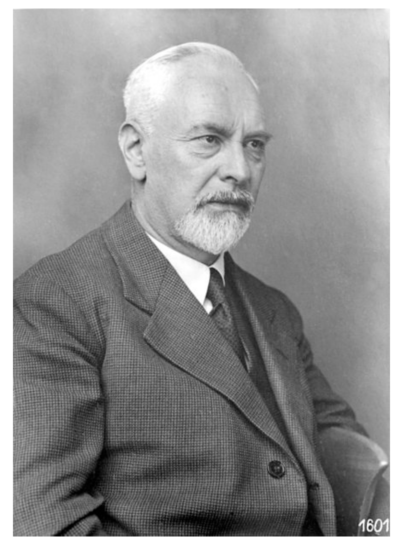
turbulent flow



$$S_b = \rho g h \sin\alpha = \text{cst} \times \rho \bar{u}^2$$

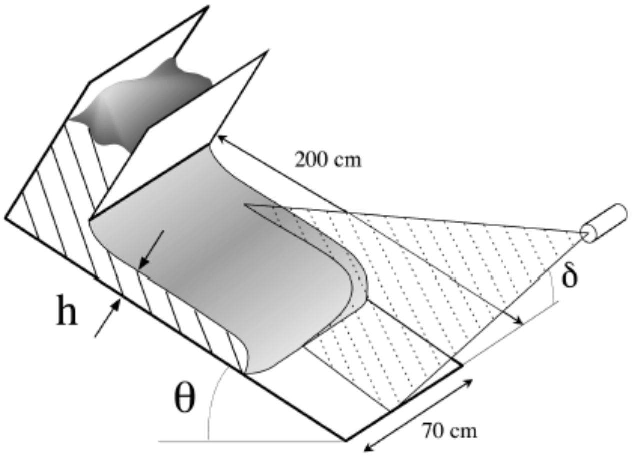
$$\text{cst} = \left(u^* / \bar{u}\right)^2 \approx 3\ 10^{-3}$$

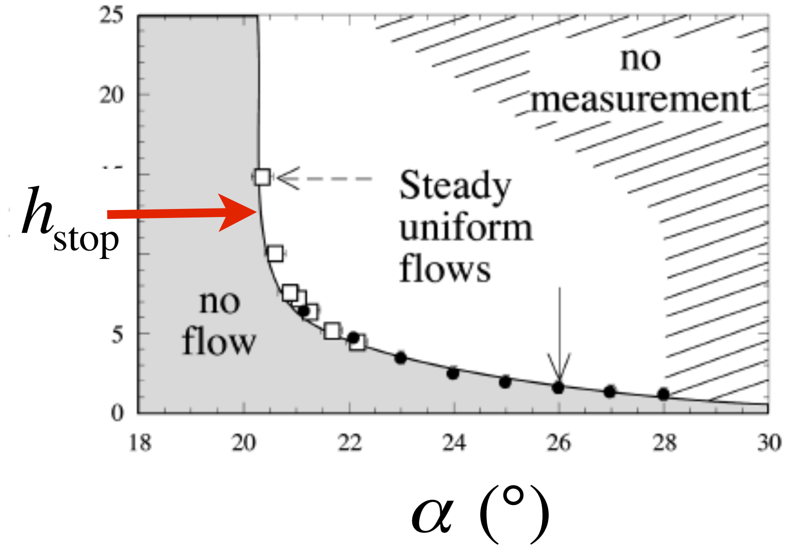
$$\frac{\dot{m}}{W} = \rho \bar{u} h = \rho \sqrt{\frac{g \sin\alpha}{\text{cst}}} h^{3/2}$$



Ludwig Prandtl

Shallow flows, far sidewalls, bumpy base

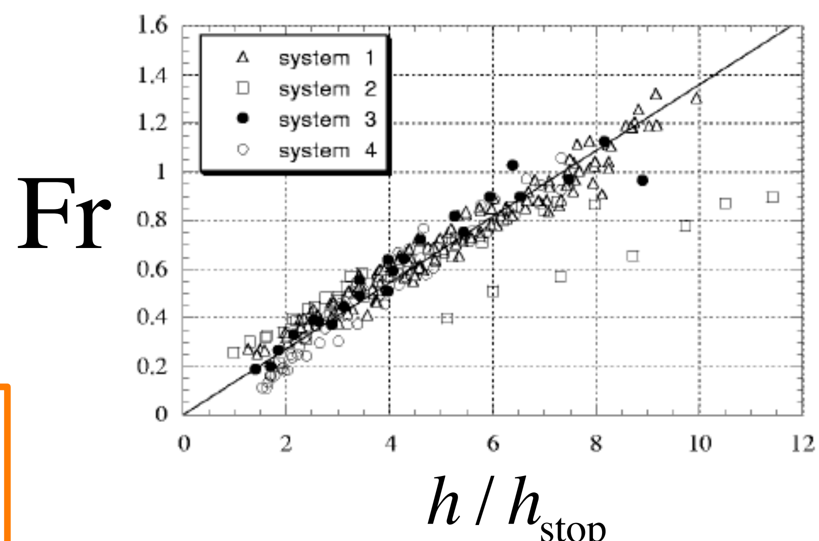




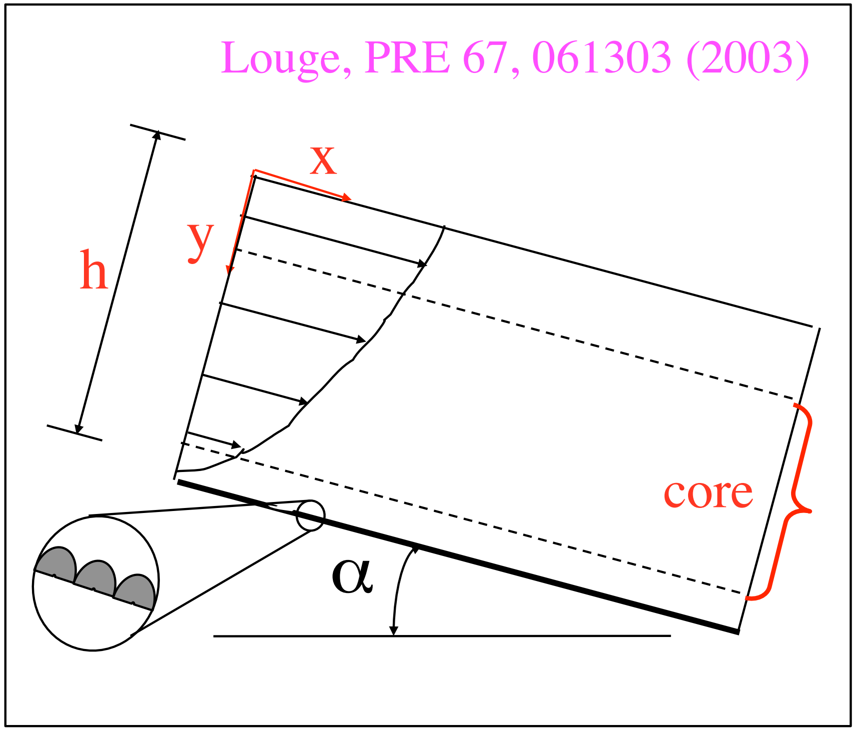
Pouliquen, Phys. Fluids 1999

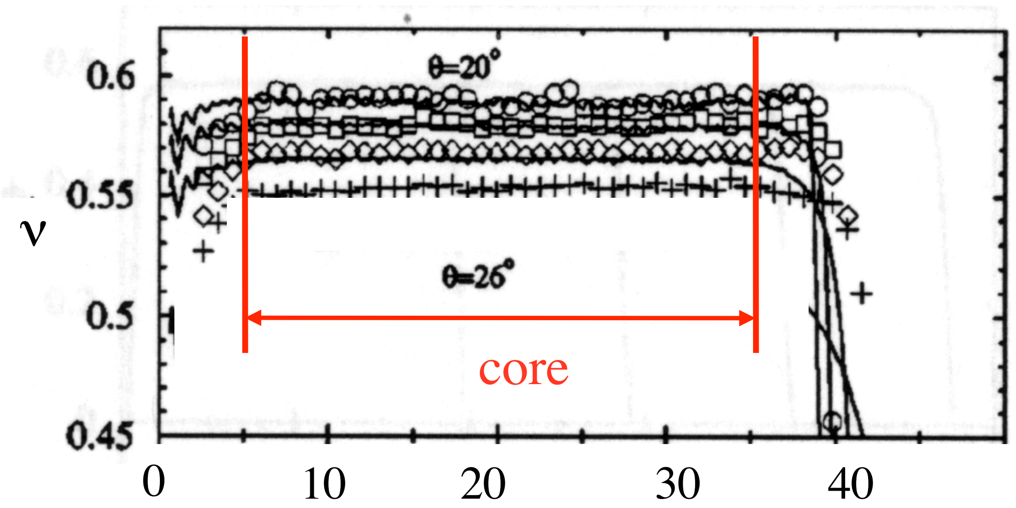
$$\frac{\dot{m}}{W} \propto \text{cst}\, \rho_s \bar{v} \frac{\sqrt{g \cos\alpha}}{h_{\text{stop}}} h^{5/2}$$

$$\text{Fr} \equiv \frac{\bar{u}}{\sqrt{N/\rho}} = \frac{\bar{u}}{\sqrt{gh\cos\alpha}}$$



Shallow flows, far sidewalls, bumpy base





Silbert, et al. PRE 2001

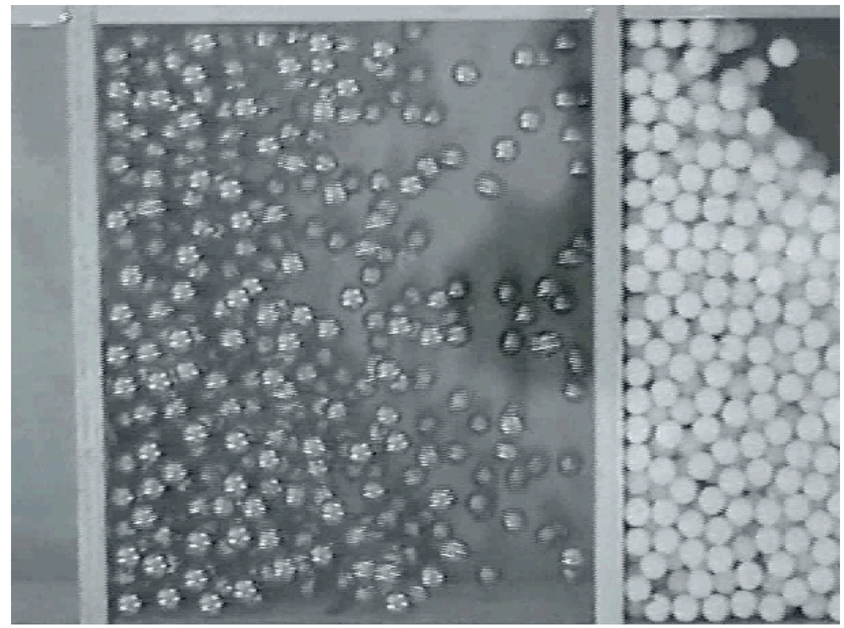
$$20^\circ < \alpha < 26^\circ$$

$$0.595 > \nu > 0.545$$

$$\frac{\dot{m}}{W} \propto \mathrm{cst}\, \rho_s \bar{\nu} \frac{\sqrt{g\cos\alpha}}{h_{\mathrm{stop}}} h^{5/2}$$

Leo Silbert

Granular temperature



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"temperature" $T = \frac{1}{3}\,\overline{u'_i\,u'_i}$

fluctuation velocity u'_i

Transport coefficients

Stuart Savage

James Jenkins

viscosity $$\mu = \rho_s d \sqrt{T} f_1(\nu; g_{12})$$

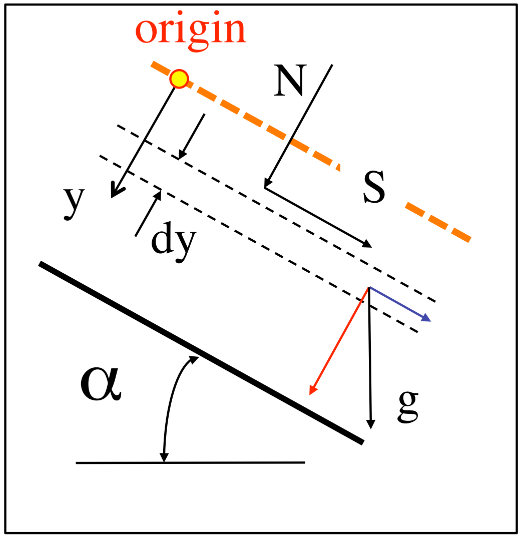
conductivity $$\kappa = \rho_s d \sqrt{T} f_2(\nu; g_{12})$$

dissipation $$\gamma = f_3(\nu; g_{12}; e)\rho_s T^{3/2} / d$$

equation of state $$N = f_4(\nu; g_{12})\rho_s T$$

Kinetic theory cartoon

Ralph Bagnold



bumpy base

viscosity

$$S = \rho_s \nu g y \sin\alpha = -f_1 \rho_s d \sqrt{T}\,\frac{du}{dy}$$

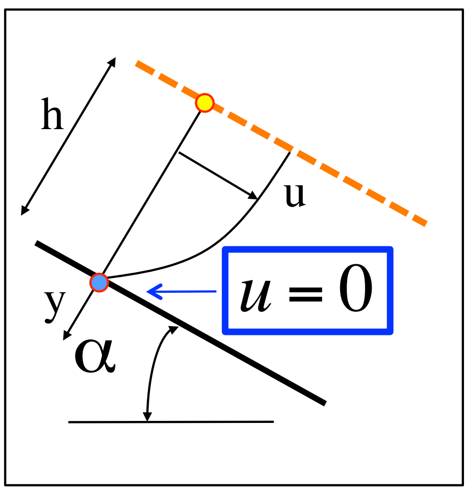
$f_1(\nu; g_{12})$

T = "granular temperature"

$f_4(\nu; g_{12})$

$$N = \rho_s \nu g y \cos\alpha = f_4 \rho_s T$$

equation of state



$$u = \frac{2}{3}\left(\frac{\sqrt{\nu f_4}}{f_1}\tan\alpha\right)\frac{\sqrt{g\cos\alpha}}{d}\left(h^{3/2} - y^{3/2}\right)$$

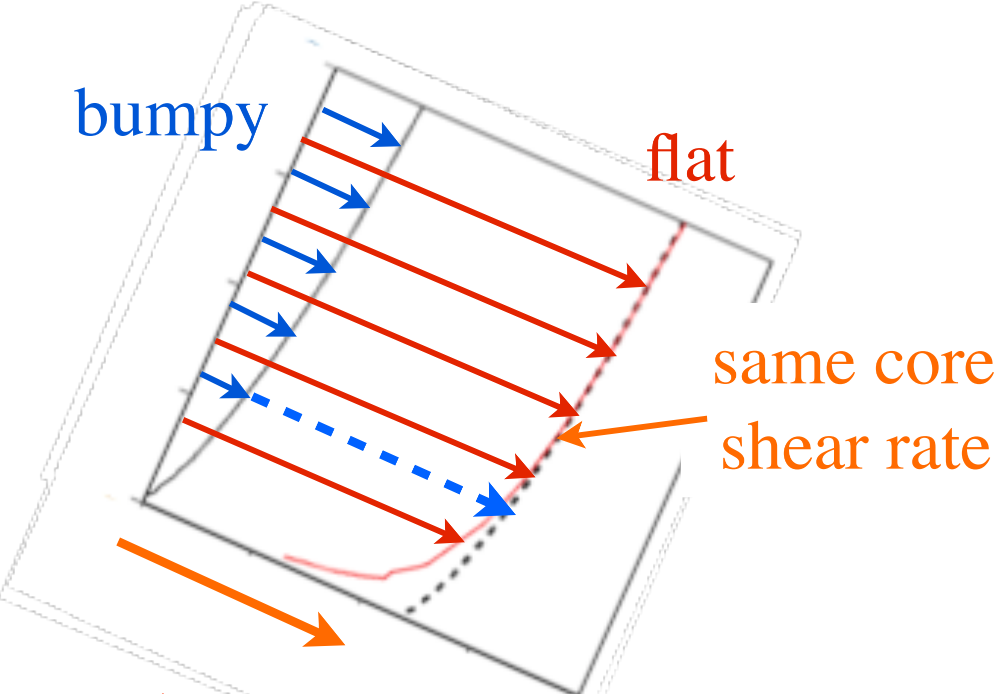
"Bagnold profile"

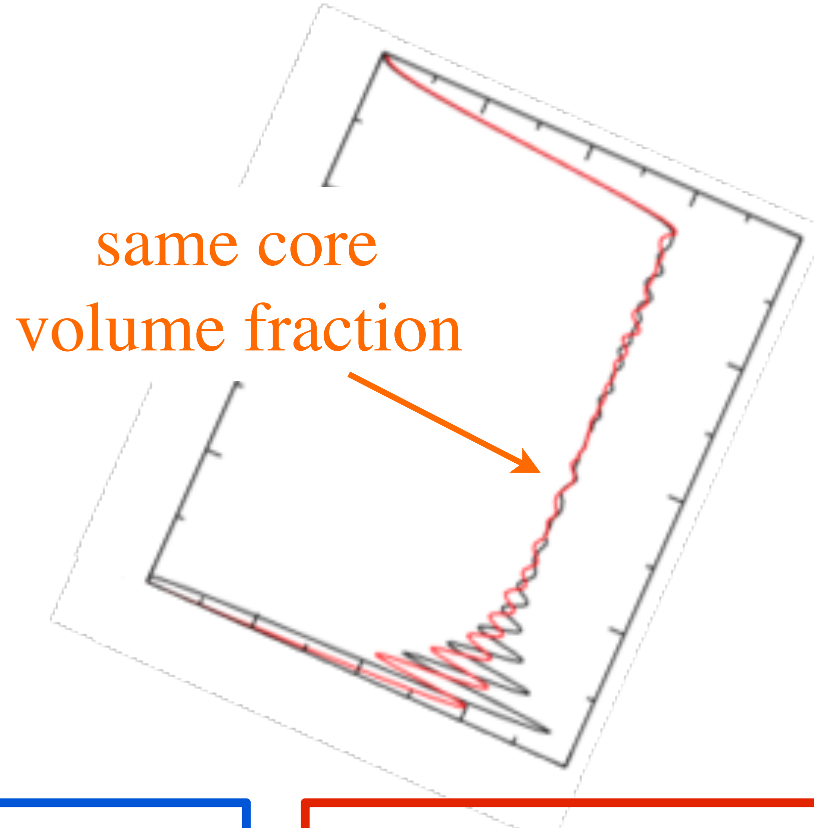
I

$$\frac{\dot{m}}{W} = \frac{2}{5}\,\frac{\nu^{3/2} f_4^{1/2}}{f_1}\,\frac{\rho_s g^{1/2}}{d}\,\frac{\sin\alpha}{\sqrt{\cos\alpha}}\,h^{5/2}$$

Flat, frictional base

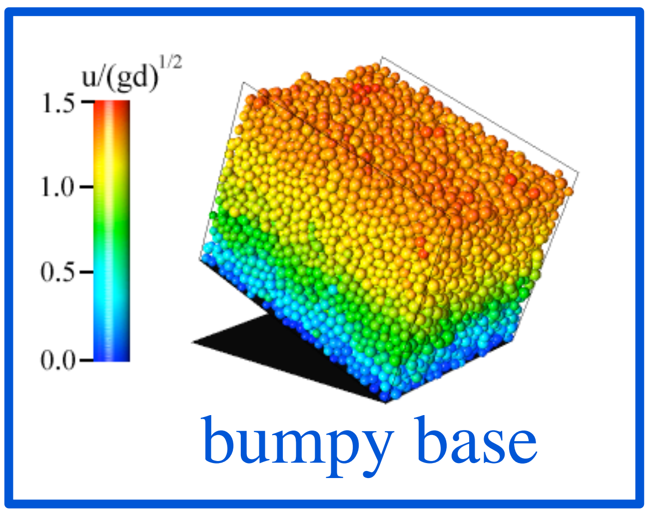
Delannay, et al, Nature Mat. 2007

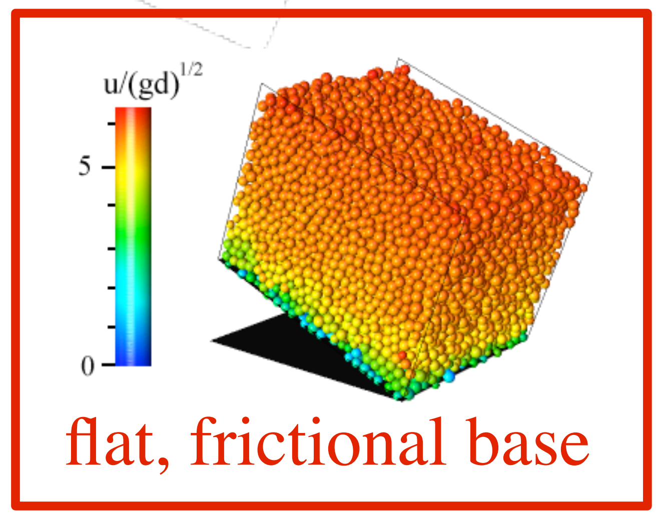




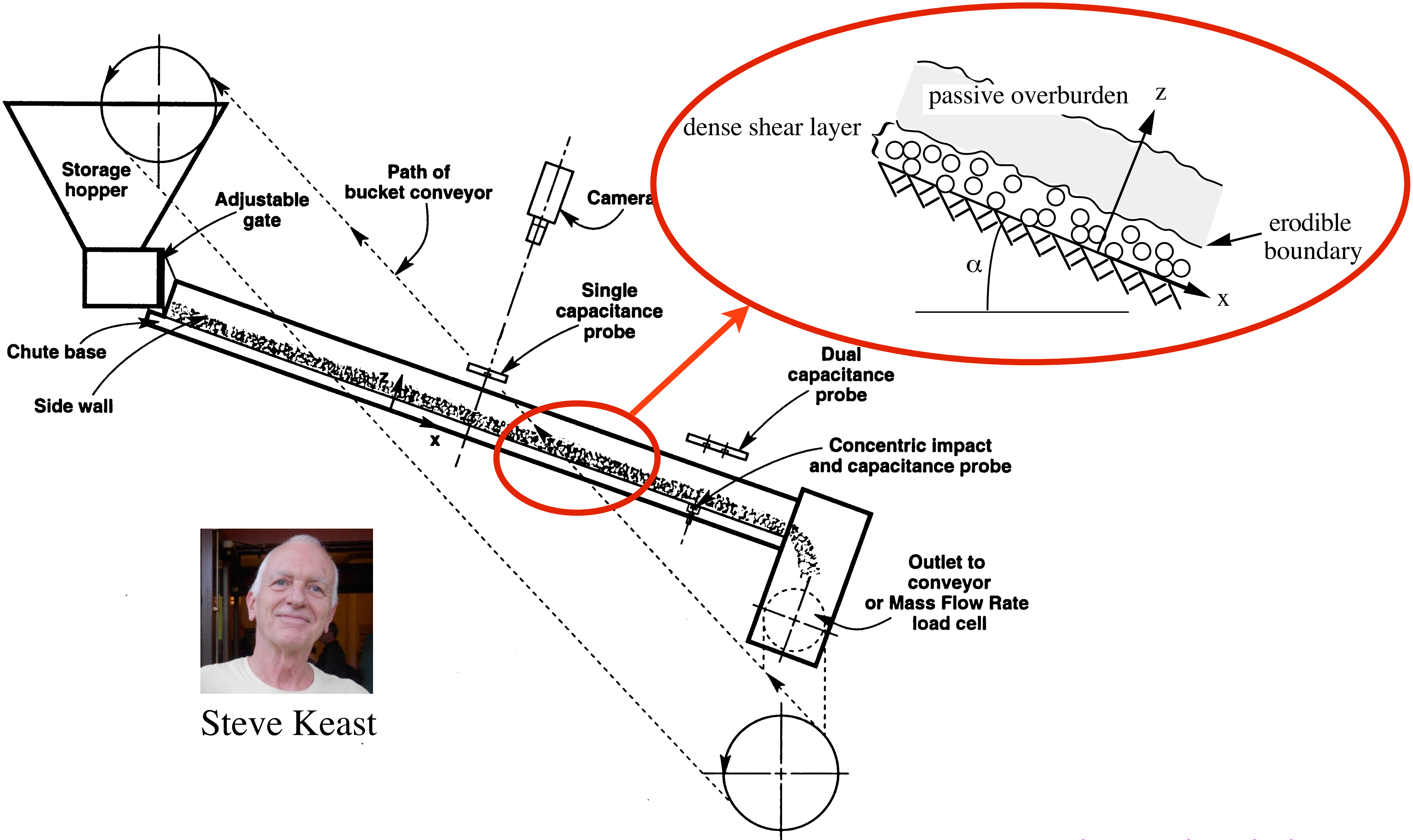
large slip
at the bottom

Simulations:
Nicolas Taberlet





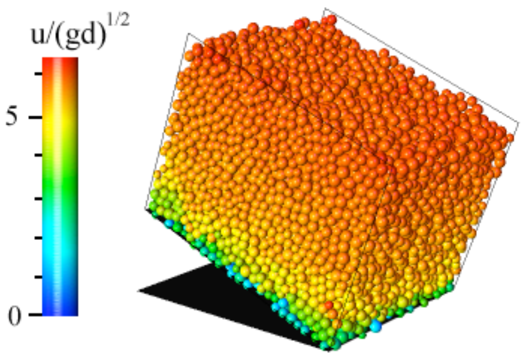
Flat, frictional base experiments



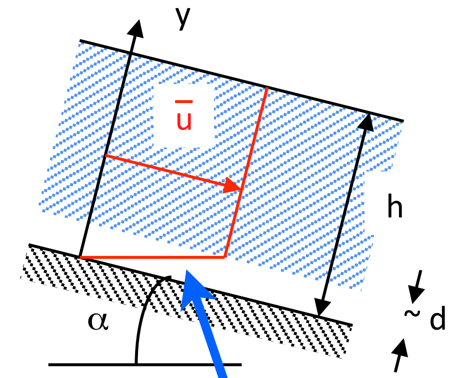
Steve Keast

Louge and Keast, Phys. Fluids 2001

Flat, frictional base cartoon



Louge and Keast, Phys. Fluids 2001



$$T_0 \sim \frac{\bar{\nu}}{f_4(\nu_0)} g h \cos\alpha$$

$$S \sim \rho_s g h \bar{\nu} \sin\alpha \sim f_1(\nu_0)\rho_s d\sqrt{T_0}\frac{\bar{u}}{d}$$

$$\mathrm{Fr} = \frac{\bar{u}}{\sqrt{gh\cos\alpha}} \sim \left(\frac{\bar{\nu}^{1/2} f_4^{1/2}}{f_1}\right)\tan\alpha$$

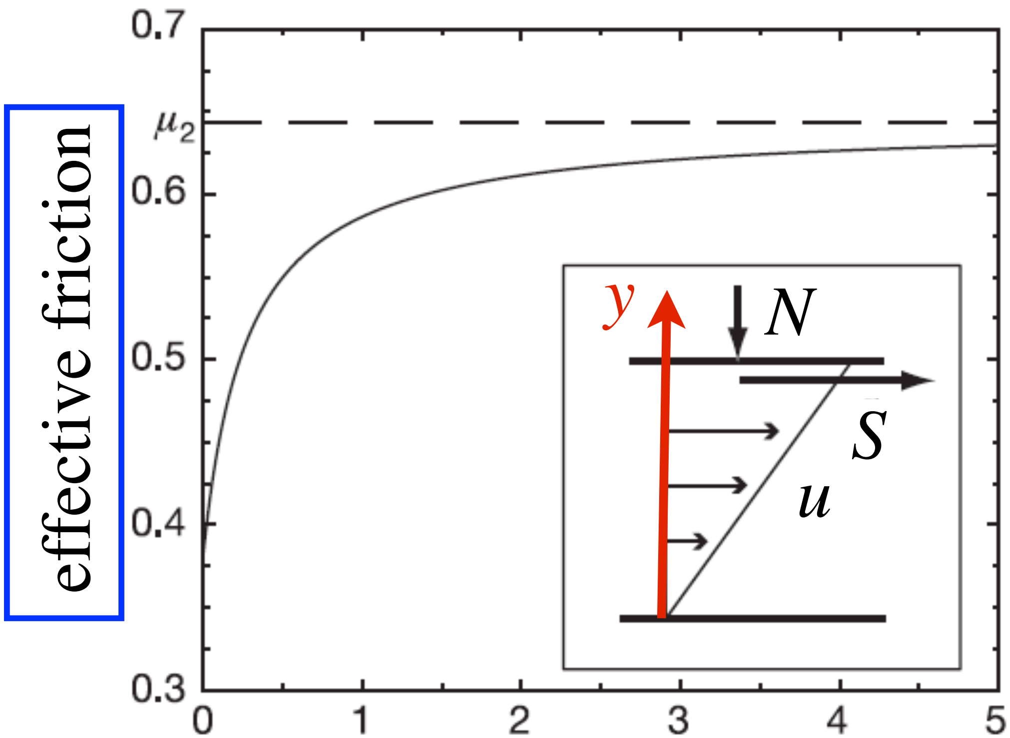
$$\frac{\dot{m}}{W} \sim \rho_s g^{1/2} \frac{\bar{\nu}^{3/2}\sin\alpha}{\sqrt{\cos\alpha}}\left(\frac{f_4(\nu_0)^{1/2}}{f_1(\nu_0)}\right) h^{3/2}$$

$T_0,\ \nu_0$

basal quantities

length scale

Inertial number



$$I \equiv \frac{|\mathrm{d}u/\mathrm{d}y|\ d}{\sqrt{N/\rho_s \nu}}$$

da Cruz, et al PRE 2005

Jop, Forterre & Pouliquen Nature 2006

GDR MiDi, Eur. Phys. J. E 2003

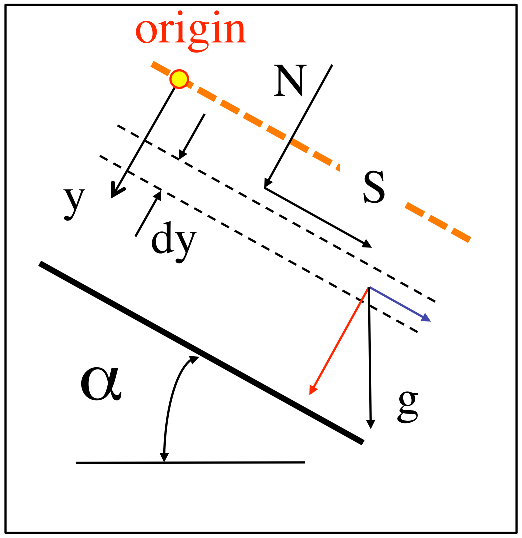
Chevoir, et al Powder Tech. 2009

Kamrin & Koval 2012

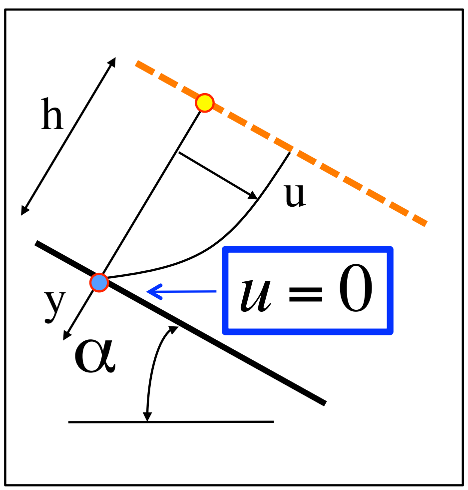
Bouzid et al 2013

Inertial number

Ralph Bagnold



bumpy base



$$u = \frac{2}{3}\left(\frac{\sqrt{\nu f_4}}{f_1}\tan\alpha\right)\frac{\sqrt{g\cos\alpha}}{d}\left(h^{3/2} - y^{3/2}\right)$$

“Bagnold profile”

I

“Non-local” extension

Kamrin & Koval 2012
Bouzid et al 2013

$$\mu = \frac{|\mathrm{d}u / \mathrm{d}y|}{f}$$

“fluidity”

$$\nabla^2 f = \frac{f - f_0}{\xi^2}$$

local fluidity

length scale

Accounts for long-range interactions and behavior at low shear rates.

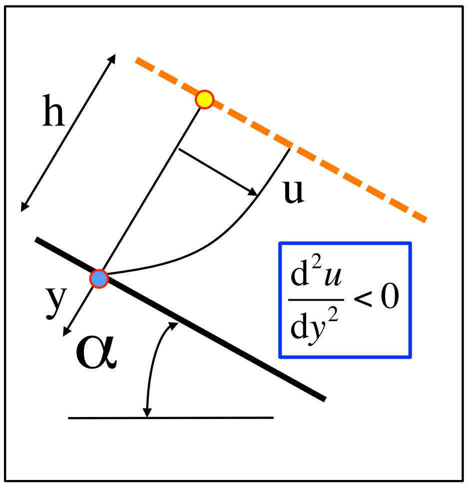
ξ and f_0 empirical functions

Boundary conditions?

Profile concavity and viscosity

$$S = \rho_s \nu g y \sin\alpha = -\mu \frac{\mathrm{d}u}{\mathrm{d}y}$$

$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} = -\rho_s g \sin\alpha \left(\frac{\nu y}{\mu}\right)\left[\frac{\nu'}{\nu} + \frac{1}{y} - \frac{\mu'}{\mu}\right]$$



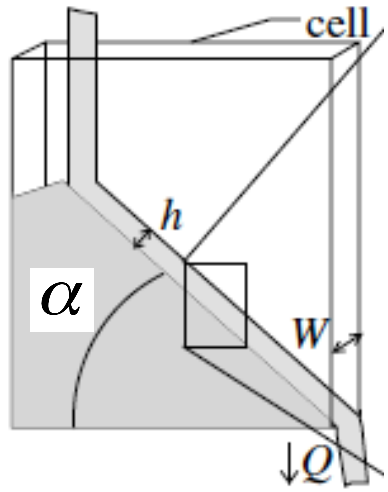
$$\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} < 0 \Leftrightarrow \frac{\mathrm{d}\ln\mu}{\mathrm{d}\ln y} < \frac{\mathrm{d}\ln\nu}{\mathrm{d}\ln y} + 1$$

bumpy base $\quad \frac{\mathrm{d}\ln\mu}{\mathrm{d}\ln y} = \frac{1}{2} \qquad \frac{\mathrm{d}\ln\nu}{\mathrm{d}\ln y} = 0$

Moderate increase in viscosity with depth for core flows over a bumpy base

Inverted concavity for a soft base

Sidewall-stabilized heap



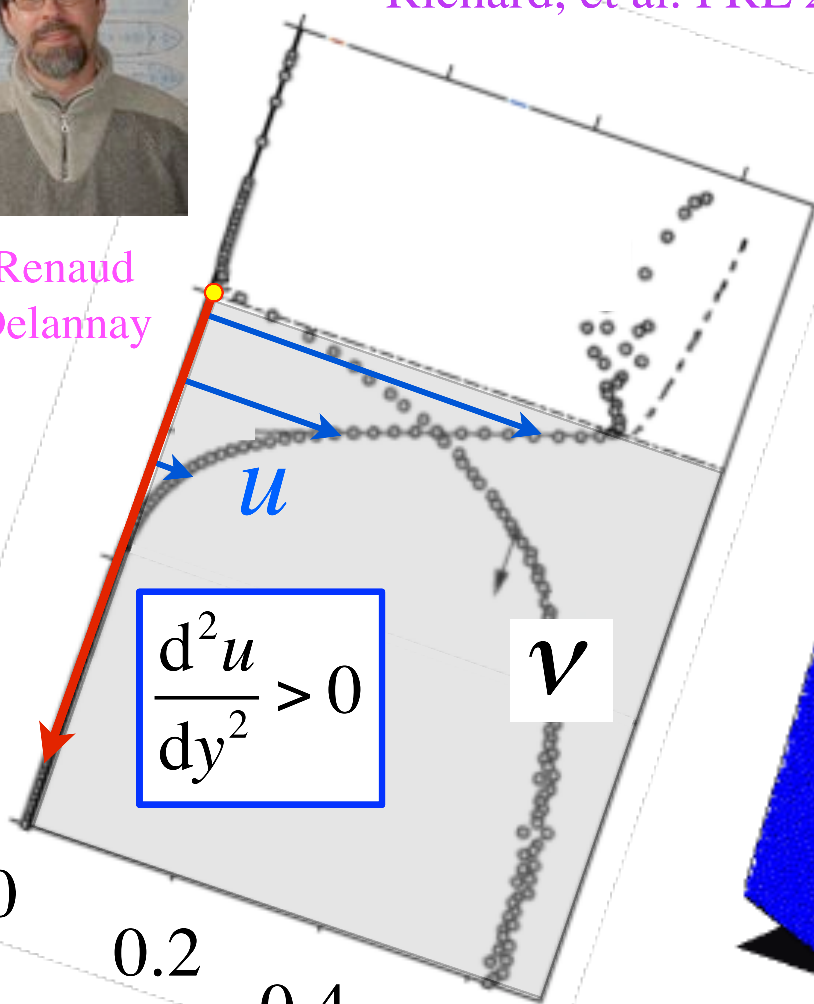
$$\frac{d^2u}{dy^2} > 0 \Leftrightarrow \frac{d\ln\mu}{d\ln y} > \frac{d\ln\nu}{d\ln y} + 1$$

Rapid increase in volume fraction and viscosity with depth.

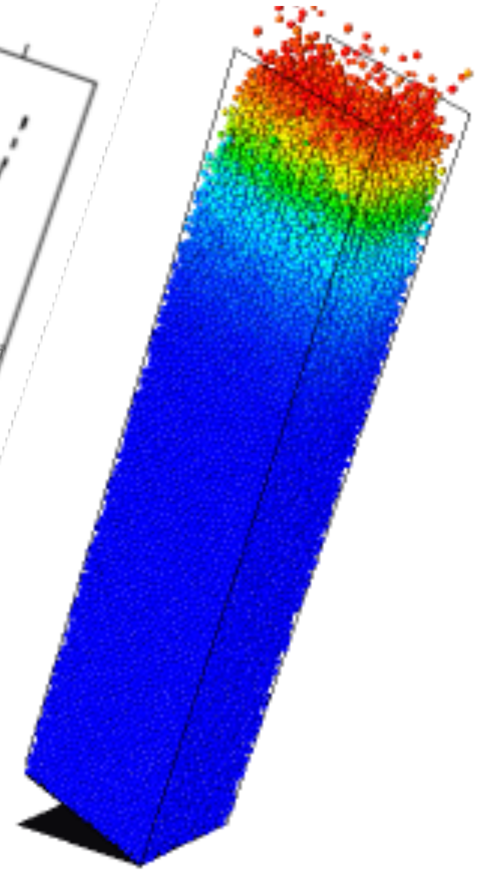
Renaud Delannay

Louge, et al. Powders & Grains 2005

Richard, et al. PRL 2008



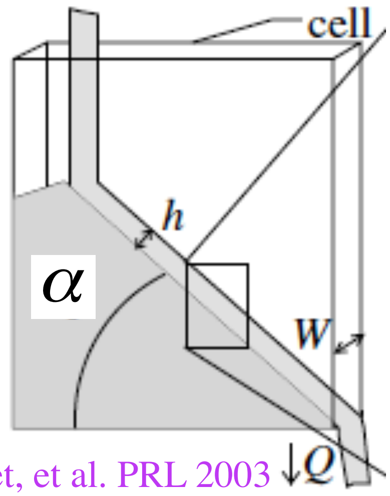
$$\frac{d^2u}{dy^2} > 0$$



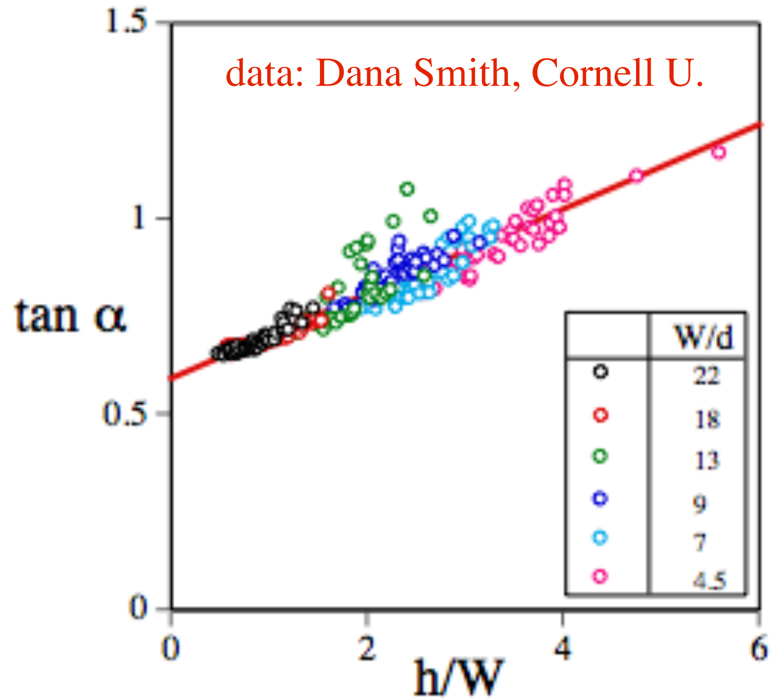
Taberlet, et al. PRL 2003

Role of side walls

Sidewall-stabilized heap



Taberlet, et al. PRL 2003

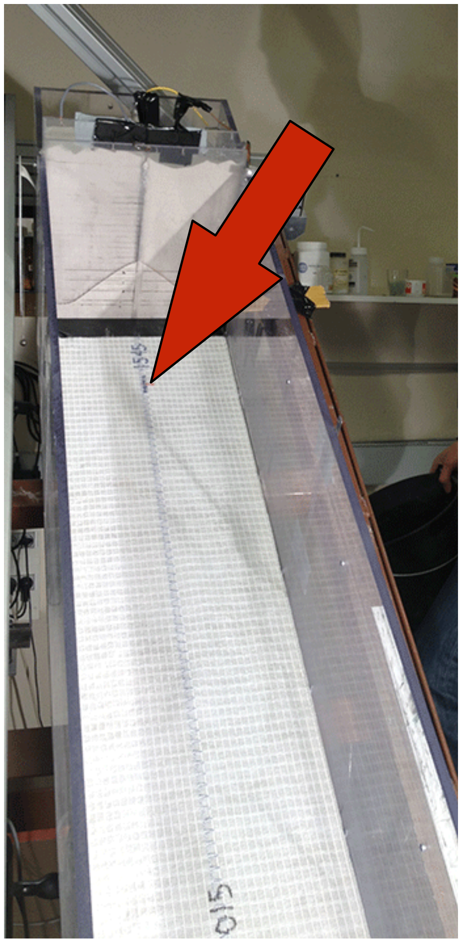


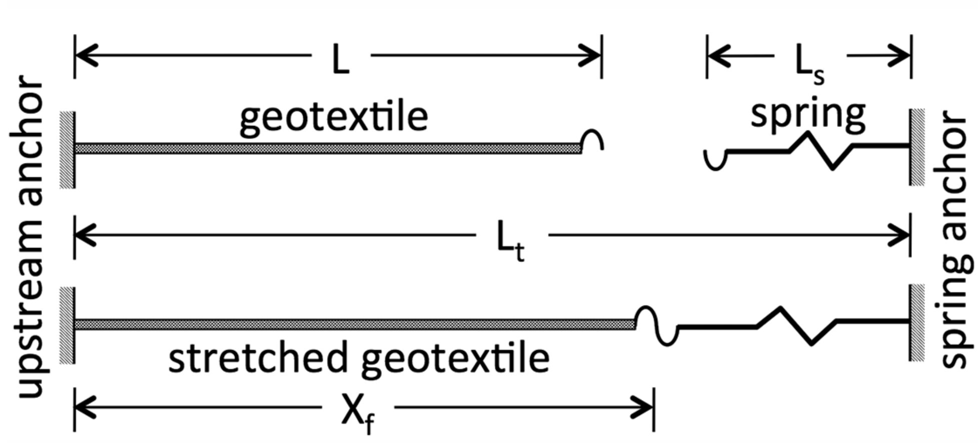
$$\tan\alpha = \mu_w\left(\frac{h}{W}\right) + \tan\alpha_{\min}$$

What about an erodible base with far side walls?

What about
an erodible base
with
far side walls?

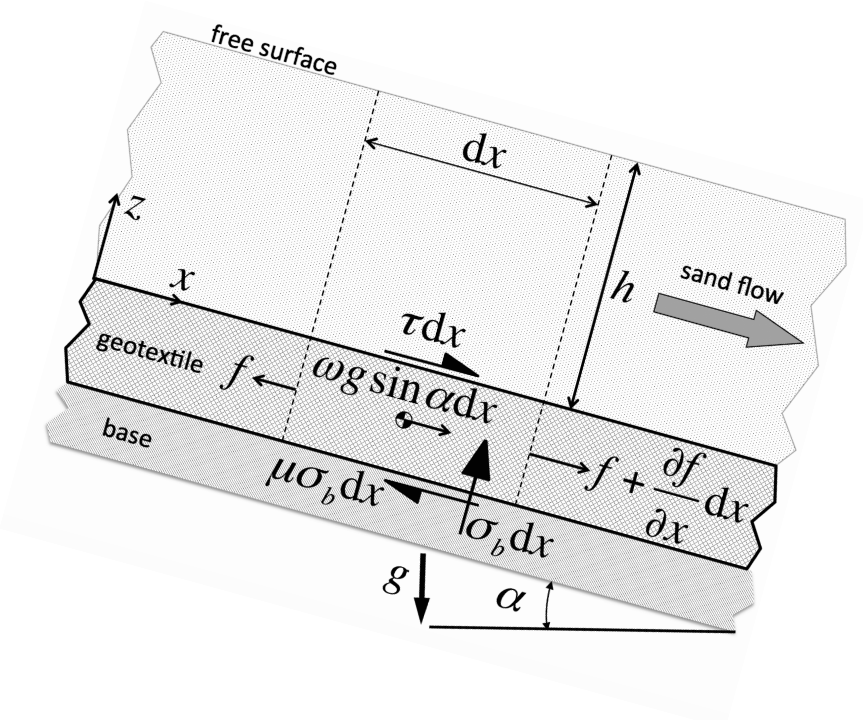
Fiber-Bragg-grating geotextile

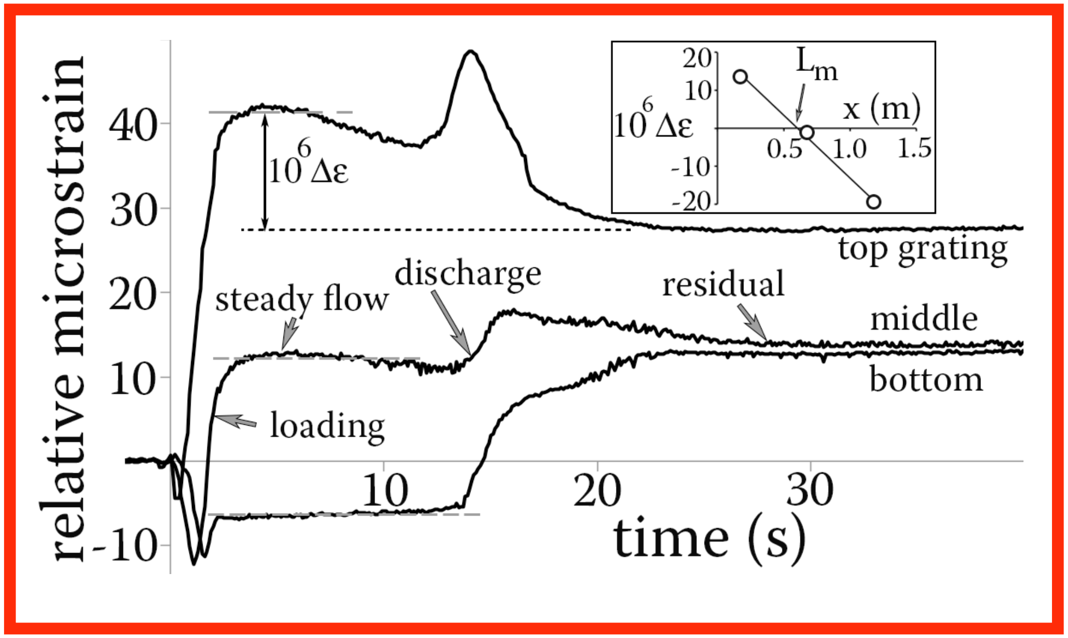




ML, A. Valance, P. Lancelot, R. Delannay, O. Artières, PRE 92, 022204 (2015)

Fiber-Bragg-grating geotextile

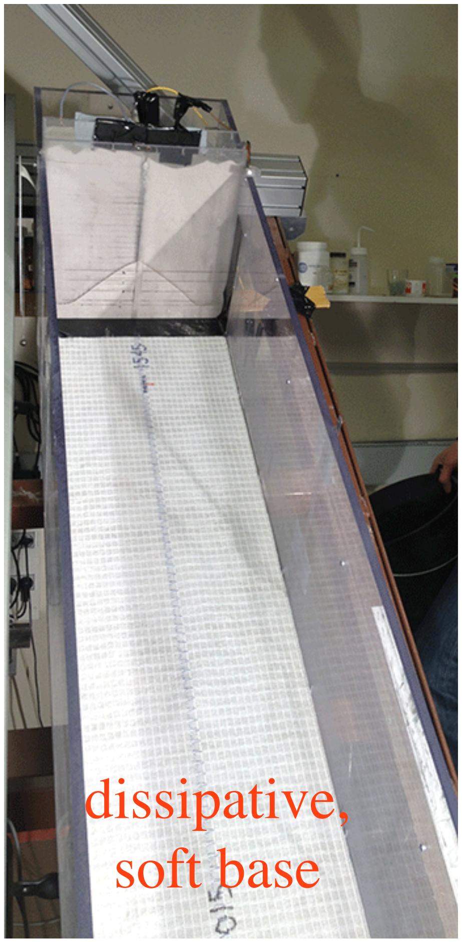




$$\Delta\varepsilon = \frac{(\tan\alpha - \mu)}{\kappa} g\cos\alpha\left(H^{\dagger} - H^{\dagger}_{\text{stop}}\right)\left(L_m - x\right)$$

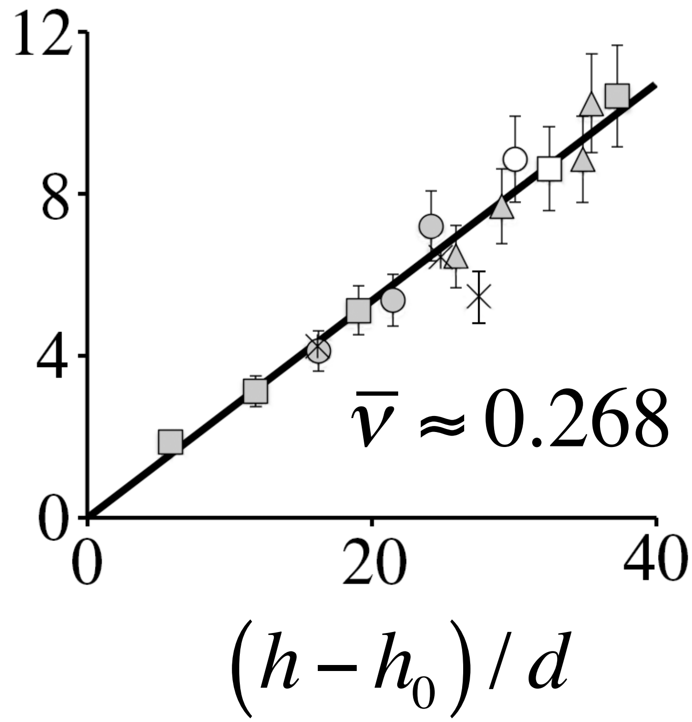
$$\Delta H^{\dagger} \equiv H^{\dagger} - H^{\dagger}_{\text{stop}} = \int_{h_0}^{h} \nu \frac{\mathrm{dy}}{d}$$

Erodible base



low mean volume fraction

$$\Delta H^{\dagger} = \int_{h_0}^{h} \nu \frac{\mathrm{d}y}{d}$$



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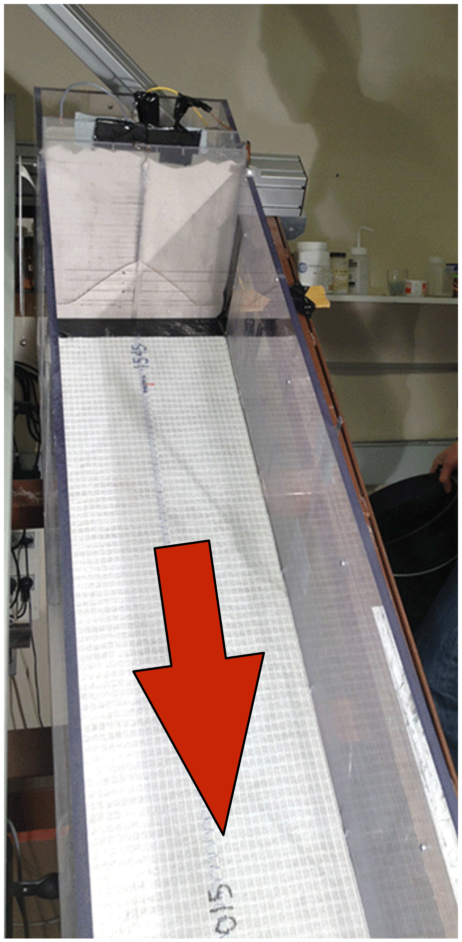
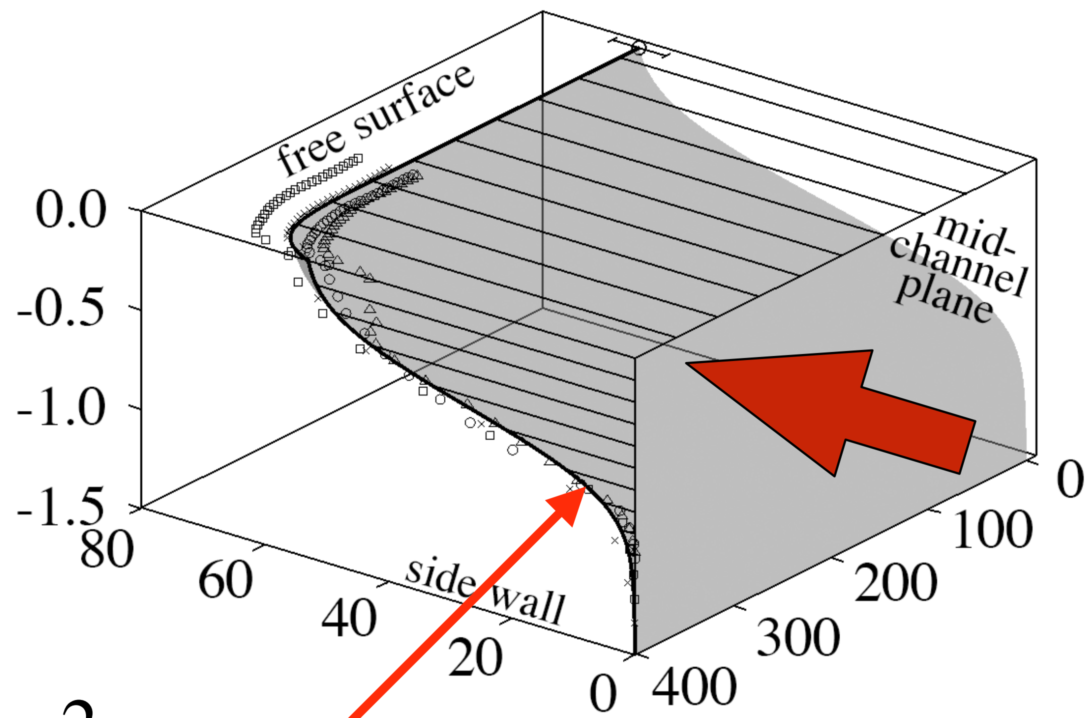
Erodible base



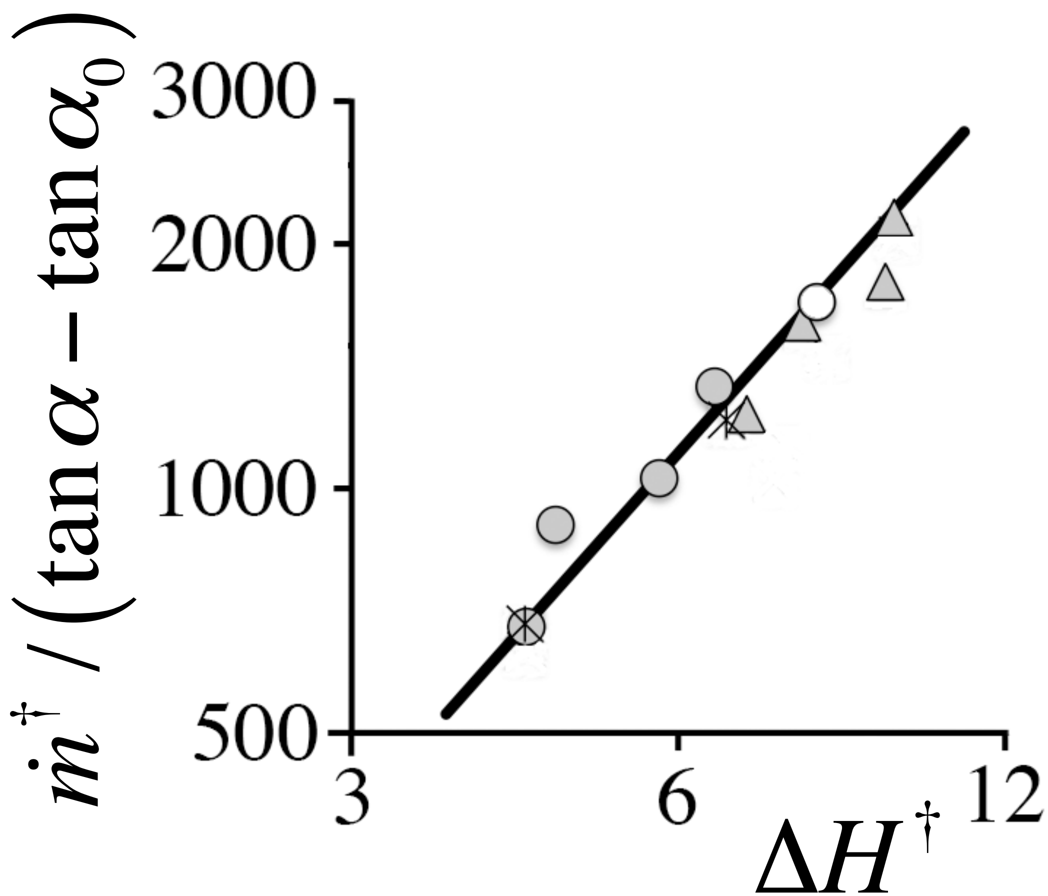


$$\frac{d^2u}{dy^2} > 0$$

ML, A. Valance, P. Lancelot, R. Delannay, O. Artières, PRE 92, 022204 (2015)

Erodible base

$$\dot{m} \propto \Delta H^{3/2}(\tan\alpha - \tan\alpha_0)$$



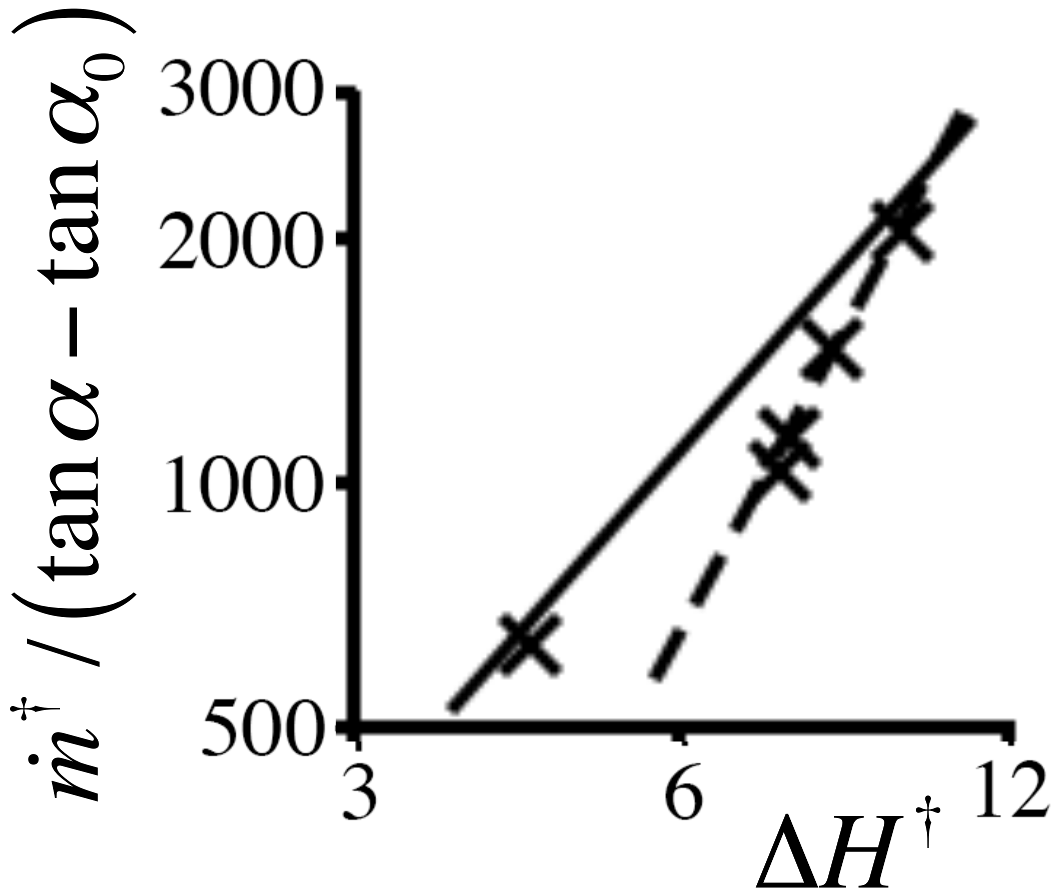
High angles $\alpha_r < \alpha < \alpha_a$

like a flat wall

Erodible base

$$\dot{m} \propto \Delta H^{5/2}\left(\tan\alpha - \tan\alpha_0\right)$$

forced-fed flows on shallow slopes



like a bumpy wall

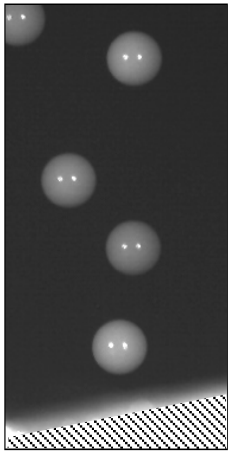
Low angles $\alpha_0 < \alpha < \alpha_r$

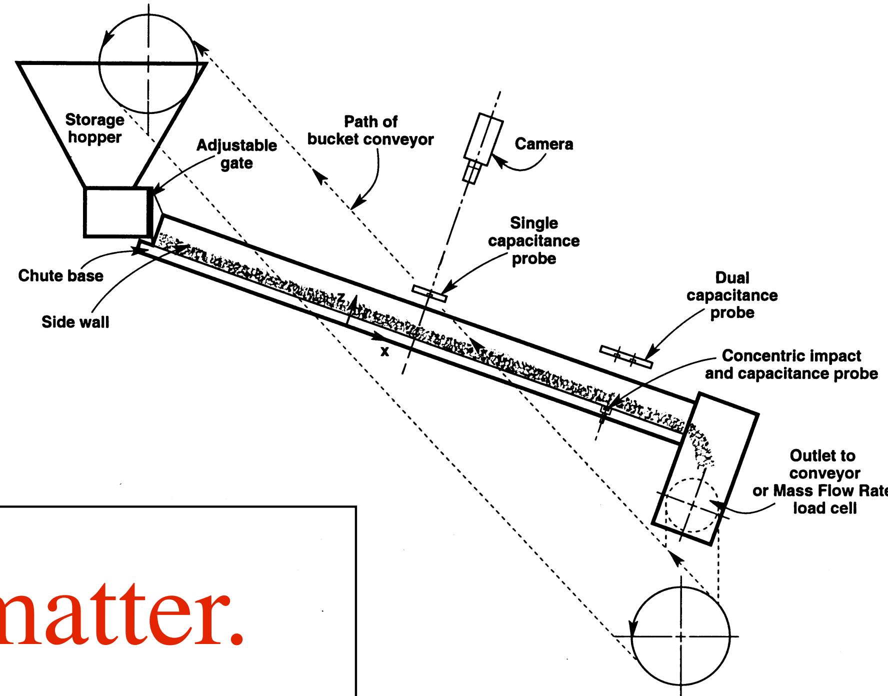
Mass flow rate versus flowing depth

flow type	n
soft, dissipative base $> \alpha_r$	3/2
Newtonian fluid, turbulent	3/2
flat, frictional base	3/2
soft, dissipative base $< \alpha_r$	5/2
core over a bumpy base	5/2
Newtonian fluid, laminar	3

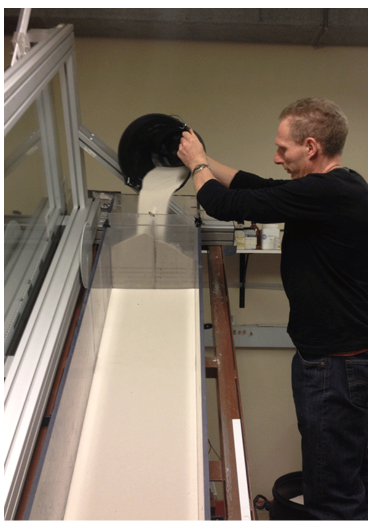
$$\frac{\dot{m}}{W} \propto H^n$$

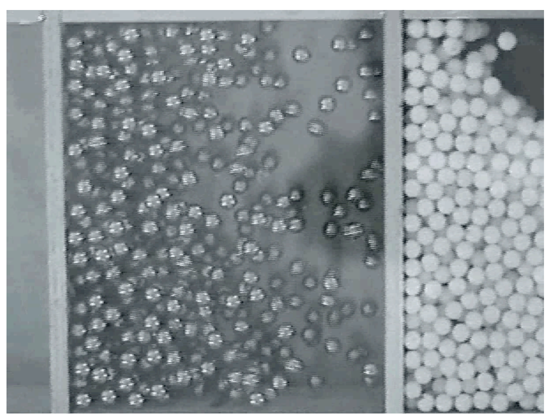
channel width W,
flowing depth H





Boundaries matter.





http://grainflowresearch.mae.cornell.edu