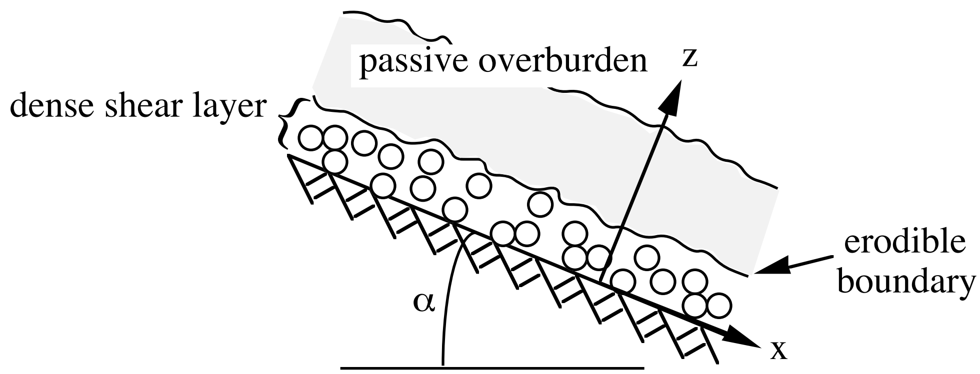
Hydraulic Model - Dense Flows

mass holdup $H^\dagger$, mean velocity $\bar{u}$ and volume fraction $\bar{\nu}$, gravity g, inclination α, grain diameter D, effective friction μ_{eff}



Mass and momentum balances

$$\frac{\partial H^\dagger}{\partial t} + \frac{\partial(\bar{u}\, H^\dagger)}{\partial x} = 0$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \frac{gD\cos\alpha}{\bar{\nu}}\,\frac{\partial H^\dagger}{\partial x} = g\cos\alpha\,(\tan\alpha - \mu_{eff})$$

Calculate μ_{eff} from local balance equations

Local solution

mass holdup $H^{\dagger}$, basal volume fraction ν_0, temperature T_0, and velocity u_0
enduring friction μ_E, impulsive friction μ_I, normal restitution e_I

Equations at the base

Shear stress: $S = S_I + S_E$

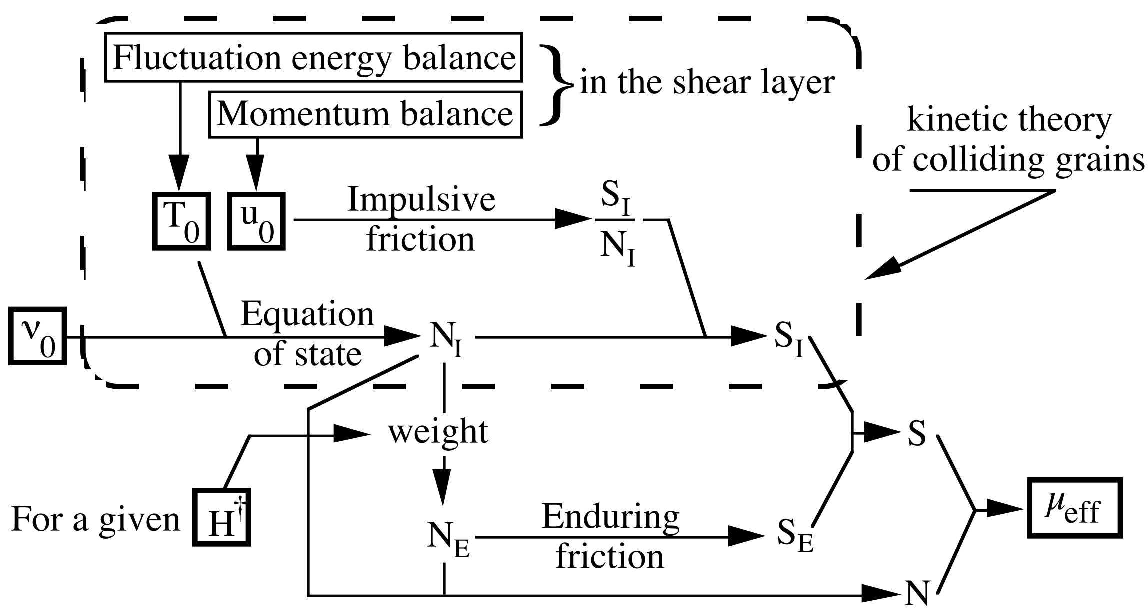
Weight: $N = N_I + N_E = \rho_s g D\, H^{\dagger} \cos\alpha$

Enduring friction: $S_E/N_E = \mu_E$

Impulsive friction: $S_I/N_I = f(u_0/\sqrt{3T_0}\,)$

Equation of state: $N_I = (1/2)\ \rho_s\ \nu_0\ g_{12}(\nu_0)\ (1+e_I)\ T_0$

Solution procedure



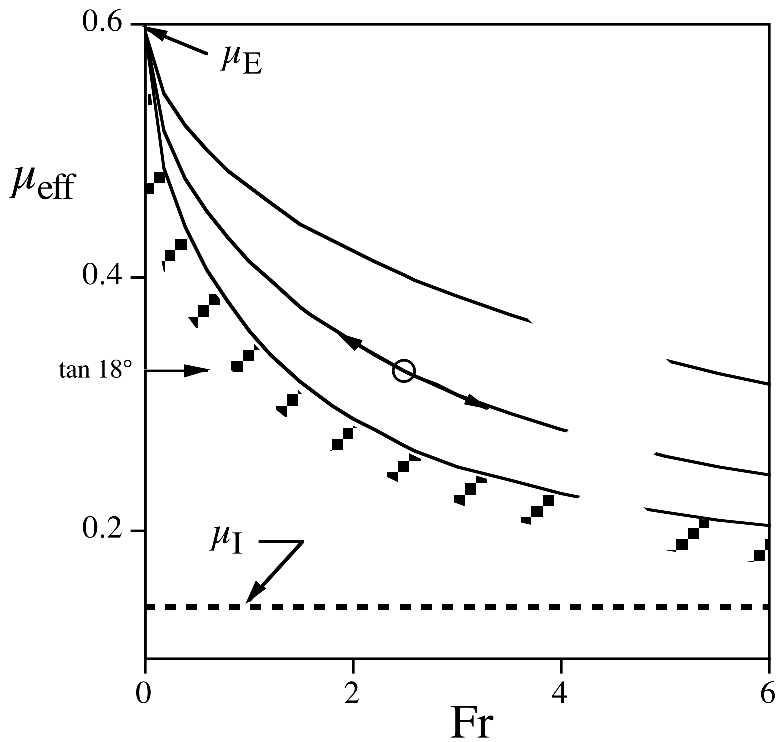
It remains to evaluate ν_0.

Effective friction

$$\mu_{eff} = \mu_E - \nu_0\, g_{12}(\nu_0)\, \frac{(1+e_I)}{2}\; \frac{(\mu_E - S_I/N_I)}{h}\; Fr^2$$

with

$$Fr \equiv \frac{\bar{u}/\sqrt{gD}}{\sqrt{H^{\dagger}\cos\alpha}} \quad \text{and} \quad h \equiv \bar{u}^2/T_0$$



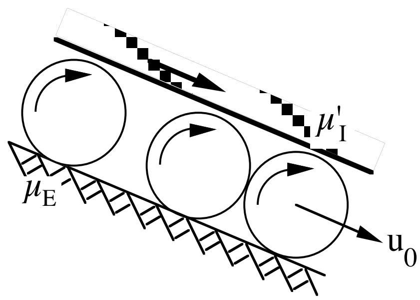
Collisions "lubricate" the flow.

If ν_0 stayed constant,
then the flow would be unstable.

Closure of ν_0 - Angular momentum balance

At steady-state, the flow adjusts ν_0 to balance production $\mathbb{P}(\omega)$ = dissipation $\mathbb{D}(\omega)$

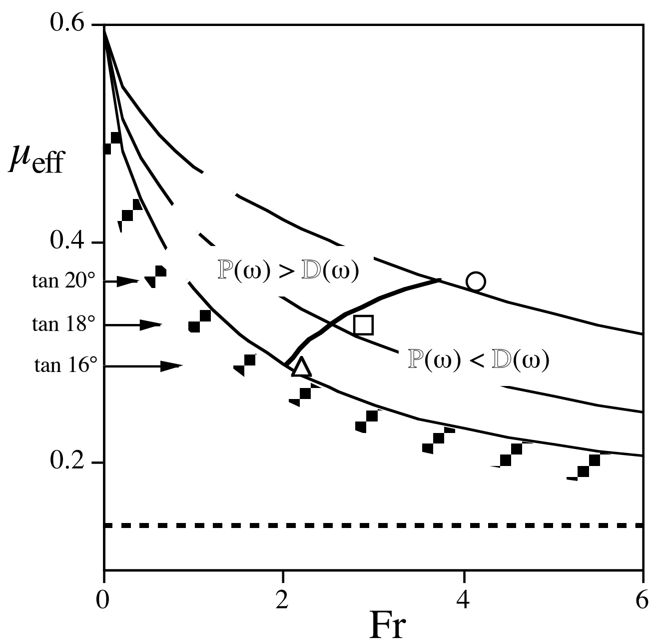
$$\mathbb{P} = (5\ g \cos\alpha/D)\ [(H/D)\ (\mu_E + \mu'_I) - \mu'_I]$$



Collisions among spheres touching the base

$$\Delta\omega = -\ (5/2D)\ (1+e'_I)\ \mu'_I\ u_0 \quad \text{at a frequency} \sim u_0\ /\ (D\pi/6\nu_0)$$

$$\Rightarrow \quad \mathbb{D} \sim (5/2D^2)\ (1+e'_I)\ \mu'_I\ u_0^2\ (6\nu_0/\pi)$$



The balance stabilizes the flow.