MEASUREMENTS OF THE COLLISION PROPERTIES OF SMALL SPHERES

by

Adam Lorenz and Michel Y. Louge†

Sibley School of Mechanical and Aerospace Engineering

Cornell University

Ithaca, New York 14853

SUMMARY

We report impact properties for the binary collisions of small spheres. The impacts are modeled in terms of three coefficients. The first is the coefficient of normal restitution. The second represents the frictional properties of the contact surfaces. The last characterizes the restitution of the tangential components of the velocity of the contact point for impacts that do not involve sliding. The coefficients are measured using a technique developed by Foerster, *et al* [*Phys. Fluids* **6**(3), 1108-1115 (1994)], who presented results for acetate and glass spheres. We now report similar data for polystyrene, acrylic and stainless steel spheres.

Theories of rapid granular flows adopt a model for a collision between a pair of spheres, then proceed to calculate the average properties of the flow using appropriate velocity distribution functions. In order to keep the corresponding integrations tractable, these theories generally employ the simplified model of Walton to treat individual collisions, rather than describing the evolution of each impact in detail [1].

† Author to whom correspondence should be addressed.

Walton's model is based on three constant impact coefficients. The velocities of spheres emerging from a collision are calculated by considering the balance of linear and angular momenta in the collision. The usual coefficient of restitution e characterizes the incomplete restitution of the normal component of **g**, the relative velocity of the spheres at the contact point,

$$\mathbf{n}.\mathbf{g}' = - e\, \mathbf{n}.\mathbf{g} \, , \qquad (1)$$

where $0 \leq e \leq 1$, **n** is the unit normal vector joining the centers of the spheres at contact, and primes denote conditions after the collision.

In collisions that involve sliding, the sliding is assumed to be resisted by Coulomb friction and the tangential and normal components of the impulse **J** are related by the coefficient of friction μ,

$$|\mathbf{n} \times \mathbf{J}| = \mu\, (\mathbf{n}.\mathbf{J}) \, , \qquad (2)$$

where $\mu \geq 0$. As the angle between **g** and **n** increases, sliding stops and (2) is replaced by

$$\mathbf{n} \times \mathbf{g}' = - \beta_0\, \mathbf{n} \times \mathbf{g} \, , \qquad (3)$$

where $0 \leq \beta_0 \leq 1$ is the tangential coefficient of restitution. In this case the point of contact is brought to rest and the collision is said to be sticking. For sticking collisions the definition of β_0 in (3) implies that some of the elastic strain energy stored in the solid during impact is recoverable, so the tangential velocity of the point of contact may be reversed. In this simple model, (2) and (3) are mutually exclusive; the point of contact is either slipping (2) or sticking (3).

Before carrying out a meaningful test of the theory in a granular flow experiment, it is essential to verify that the three impact coefficients e, μ and β_0 employed in the theory provide an adequate description of the collision and, if so, to determine their values. Unfortunately, because their measurement involves the precise control of the trajectories of small spheres, these coefficients were seldom determined in the past.

In a recent paper [2], we described an experimental apparatus that measures the collision properties of small spheres. The apparatus includes a mechanism that brings two identical particles into a collision without initial spin, and a stroboscopic setup that photographs the dynamics of their flights. The setup can also produce impacts between a single sphere and a flat plate. In that paper, we reported impact coefficients for binary collisions of cellulose acetate and glass spheres and for collisions of these spheres with a thick, smooth, flat aluminum plate. For these materials we showed that Walton's model captures the behavior of the impact over a wide range of incident angles. The objective of the present paper is to report similar measurements for polystyrene, acrylic and stainless steel spheres.

Details of the experimental apparatus and the analysis of the data are found in Foerster, *et al* (1994) [2]. The experimental technique for the polystyrene and acrylic spheres is identical to that employed previously with the acetate and glass spheres. However, the stainless steel spheres present additional difficulties. Here, because most of the stroboscopic light is specularly reflected by the shiny metal surface, the circular edges of the particles cannot be distinguished on film. Instead, two bright spots located well within the sphere's outline overwhelm the photographs. We resolve this difficulty by painting a portion of the spheres that is not involved in the collision with a fluorescent dye. The fluorescence produces bright regions that permit to distinguish clearly the circular outline of the spheres.

A convenient way to interpret the data is to follow Maw, Barber and Fawcett [3,4] and produce a plot of $\Psi_2 \equiv -(\mathbf{g}'.\mathbf{t})/(\mathbf{g}.\mathbf{n})$ versus $\Psi_1 \equiv -(\mathbf{g}.\mathbf{t})/(\mathbf{g}.\mathbf{n})$, where $\mathbf{t}$ is a unit vector located in the collision plane $(\mathbf{g},\mathbf{n})$ and tangent to both spheres. In collisions that involve gross sliding,

$$\Psi_2 = \Psi_1 - \frac{7}{2}(1+e)\,\mu\,\mathrm{sign}(\mathbf{g}.\mathbf{t}); \qquad (4)$$

and in collisions that do not,

$$\Psi_2 = -\beta_0 \Psi_1 . \tag{5}$$

Figures 2 to 3 present the experimental data in that form. The corresponding results are summarized in the Table.

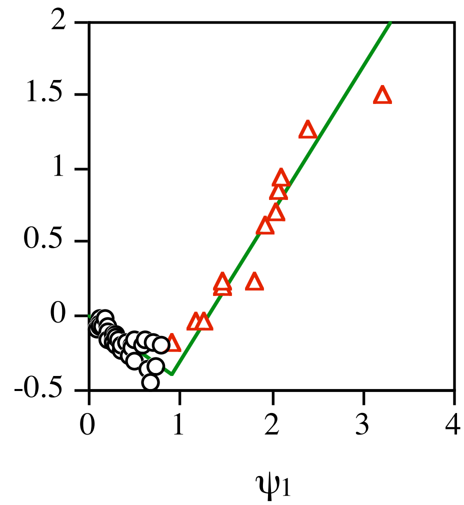

Fig. 1. Results for binary collisions of 4mm polystyrene spheres. The circles and triangles represent the sticking and slipping regime, respectively. The solid lines are least-squares fits of (4) and (5) to the data.

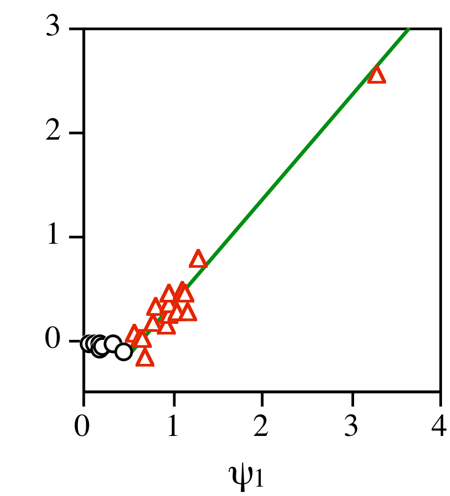

Fig. 2. Results for binary collisions of 4mm acrylic spheres. See Fig. 1 for symbols.

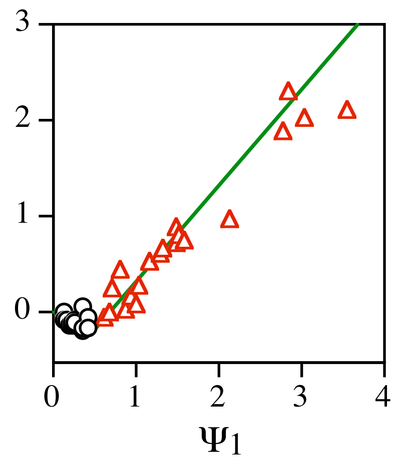

Fig. 3. Results for binary collisions of 5mm steel spheres. See Fig. 1 for symbols.

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REFERENCES

1. O.R. Walton: "Granular Solids Flow Project, Quarterly Report, January-March 1988, UCID-20297-88-1," Lawrence Livermore National Laboratory (1988).

2. S.F. Foerster, M.Y. Louge, H. Chang, and K. Allia: "Measurements of the collision properties of Small Spheres," *Phys. Fluids* **6**(3), 1108-1115 (1994).

3. N. Maw, J.R. Barber and J.N. Fawcett: "The Oblique Impact of Elastic Spheres," *Wear* **38**(1), 101-114 (1976).

4. N. Maw, J.R. Barber and J.N. Fawcett: "The Role of Elastic Tangential Compliance in Oblique Impact," *ASME J. of Lubrication Technology* **103**, 74-80 (1981).