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Impacts on a stationary tetrahedral array of spheres

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Résumé - Nous rapportons le comportement particulier des impacts obliques d'une sphère sur des sphères identiques formant un empilement tétraédrique. Nous comparons leurs coefficients de restitution normale avec ceux inhérents aux impacts avec une sphère fixée à une plaque épaisse ou avec une autre sphère. Les résultats ont trait à la formation d'ejecta dans le processus de saltation.

Abstract - We report the peculiar behavior of the oblique impacts of a single sphere on stationary like spheres arranged in a tetrahedral array. We compare the resulting kinematic coefficient of normal restitution to that in impacts on a single sphere attached to a thick plate and in binary impacts. The results provide detailed insight into the generation of ejecta in the saltation process.

Keywords – Impact, normal restitution, saltation.

1. Introduction

The saltation process responsible for the migration of desert dunes depends crucially on the dynamics of the individual impacts of an entrained grain of sand with arrays of stationary grains on the dune surface. Models of this phenomenon require the empirical input of a "splash function" that quantifies the amount and direction of the corresponding ejecta.[1]

To inform this process, we carried out controlled experiments with model spheres of Delrin in a facility designed by Foerster, et al.[2,3] The experiment releases a single sphere without spin from a known height. The ballistic trajectories of the falling sphere and that of its impact protagonist are monitored using stroboscopic photography. For simplicity, the impact is modeled using three coefficients.[4] The first is the Newtonian kinematic coefficient of normal restitution e. It characterizes the incomplete restitution of the relative velocity $\mathbf{u}$ at the contact point projected along the normal $\mathbf{n}$ joining the centers of mass of the two colliding spheres,

$$\mathbf{n}.\mathbf{u}' = - e\ \mathbf{n}.\mathbf{u}\ , \qquad (1)$$

where primes denotes values after impact. The incident angle γ between $\mathbf{u}$ and $\mathbf{n}$ characterizes the impact geometry, $\cot \gamma \equiv \mathbf{u}.\mathbf{n} / |\mathbf{u} \times \mathbf{n}|$. Because impacts occur when $\mathbf{u}.\mathbf{n} \leq 0$, this angle lies in the range $\pi/2 \leq \gamma \leq \pi$.

The second parameter in this model arises when grazing collisions with incident angles near $\pi/2$ involve gross sliding. For these, we assume that sliding is resisted by Coulomb friction and that the tangential and normal components of the impulse $\mathbf{J}$ are related by the coefficient of friction $\mu \geq 0$,

$$|\mathbf{n} \times \mathbf{J}| = \mu\ (\mathbf{n}.\mathbf{J})\ . \qquad (2)$$

For greater values of the incident angle, the impact is closer to head-on and it no longer involves gross sliding as parts of the contact patch are brought to rest. When γ exceeds the limiting angle γ_0, Equation (2) is replaced by

$$\mathbf{n} \times \mathbf{u}' = -\beta_0\, \mathbf{n} \times \mathbf{u}\ , \qquad (3)$$

where $\gamma_0 \equiv \pi - \tan^{-1}[7(1+e)\mu/2(1+\beta_0)]$ and β_0 is the tangential coefficient of restitution. Although crude,[3] this model is a convenient input to collisional theories of granular flows.[5]

To interpret data from an impact experiment, Maw, Barber and Fawcett[6,7] produce a plot of $\Psi_2 \equiv -(\mathbf{u}'.\mathbf{t})/(\mathbf{u}.\mathbf{n})$ versus $\Psi_1 \equiv -(\mathbf{u}.\mathbf{t})/(\mathbf{u}.\mathbf{n})$, where **t** is a unit vector located in the collision plane (**u**,**n**) and perpendicular to **n**.[2,3] In collisions of a homogeneous sphere that involves gross sliding,

$$\Psi_2 = \Psi_1 - \frac{7}{2}(1+e)\mu\ \mathrm{sign}(\mathbf{u}.\mathbf{t}); \qquad (4)$$

and in collisions that do not,

$$\Psi_2 = -\beta_0\ \Psi_1\ . \qquad (5)$$

For positive values of **u**.**t**, $\Psi_1 = |\tan\gamma|$. Similarly, $(\Psi_2/e) = \tan\gamma'$ is the tangent of the recoil angle γ' between **n** and **u**'. If the coefficients e, μ and β_0 are constant, data plotted as Ψ_2 versus Ψ_1 fall on two distinct straight lines [Eqs. (4) and (5)] that permit unambiguous identification of the sliding and sticking regimes. This paper reports a case where such a plot must be complemented by a detailed look at the dependence of e on Ψ_1.

2. Observations

In the first series of experiments, we collided two identical spheres of Delrin with different sizes (Fig. 1). As Table 1 indicates, the corresponding coefficients of restitution and friction showed no discernible effect of sphere size within experimental error. Figure 2 shows the corresponding graph of Ψ_2 versus Ψ_1 for 3.2 mm spheres.

Table 1 : *Impact parameters in binary collisions of Delrin spheres*

sphere diameter	2.4 mm	3.2 mm	4.8 mm	6.4 mm
e	0.94 ± 0.02	0.97 ± 0.02	0.97 ± 0.01	0.97 ± 0.01
μ	0.17 ± 0.07	0.19 ± 0.01	0.18 ± 0.01	0.14 ± 0.01
β_0	0.46 ± 0.01	0.44 ±0.03	0.52 ± 0.03	0.54 ± 0.1

In the next series of experiments, we impacted a falling sphere on a single like sphere firmly attached to a base with adjustable inclination (henceforth called the "single sphere" configuration, Fig. 3). Varying the latter allowed us to control the angle of incidence. As Fig. 4 shows, the corresponding coefficient of normal restitution was smaller than its binary counterpart, but there was no discernible effect of angle of incidence on this coefficient in either case.

Finally, we prepared the tetrahedral array of identical spheres sketched in Fig. 5. The bottom three spheres of this stationary array were firmly attached to one another and to the horizontal base. The top sphere was either glued to the other three (the "attached" configuration, Fig. 6) or was simply resting on them (the "free" configuration, Fig. 7) until the falling sphere struck it.

As Fig. 8 shows, the coefficients of normal restitution in the two tetrahedral configurations decrease substantially with the tangent of the angle of incidence i.e., for increasingly grazing impacts. As expected intuitively, the "free" configuration dissipates more energy than its "attached" counterpart, as little energy can return to the falling sphere once its impact protagonist has been ejected. Similarly, because both the "free" and "attached" configurations provide further paths for dissipating the incoming energy than the "single sphere" of Fig. 3, they exhibit substantially lower restitution for oblique collisions, which are likely to prevail in the saltation process. However, for head-on impacts with $\tan\gamma = 0$, the normal restitution of the "single sphere" and "attached" configurations are identical.

3. Acknowledgments

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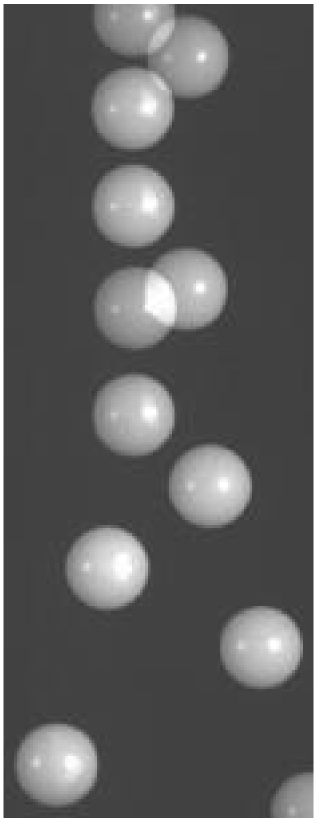

Figure 1 : *Stroboscopic photograph of the binary collision of Delrin spheres of 3.2 mm diameter.*

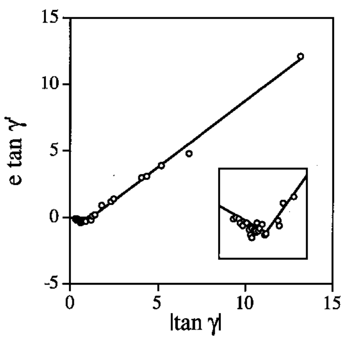

Figure 2 : *Results for the binary collisions of 3.2 mm Delrin spheres. The ordinate is* Ψ_2 *and the abscissa is* Ψ_1*. The insert shows details near the origin.*

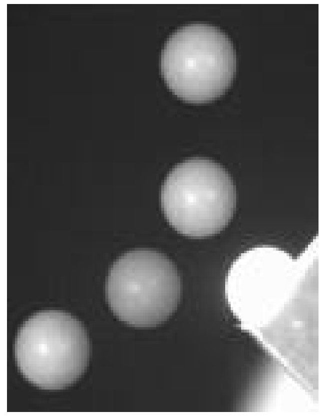

Figure 3 : *Stroboscopic photograph of the collision of a 3.2 mm Delrin spheres with a similar sphere rigidly affixed on a base.*

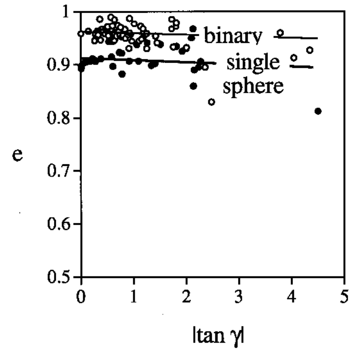

Figure 4 : *Variations of the coefficient of normal restitution with angle of incidence. The open and closed circles represent binary impacts and impacts with a single fixed sphere, respectively. The lines are visual fits to the data.*

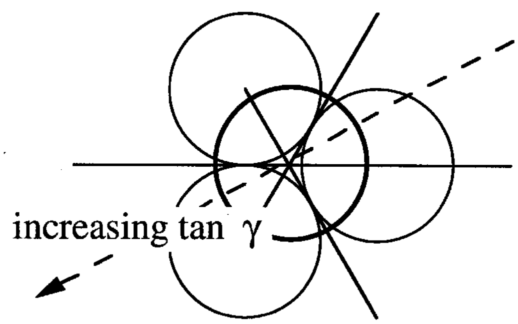

Figure 5 : *Arrangement of the stationary spheres. The heavy circle marks the position of the top sphere. The arrow points to the direction of the falling sphere after impact.*

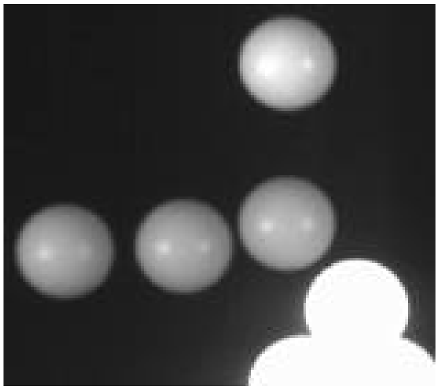

Figure 6 : *Stroboscopic photograph of the impact of a sphere on like spheres in the "attached" configuration.*

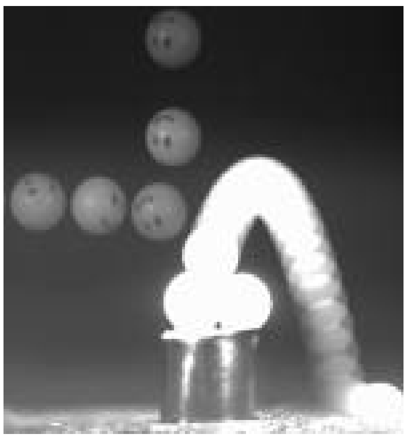

Figure 7 : *Stroboscopic photograph of the impact of a sphere on like spheres in the "free" configuration. The free sphere, originally at rest, is ejected to the right.*

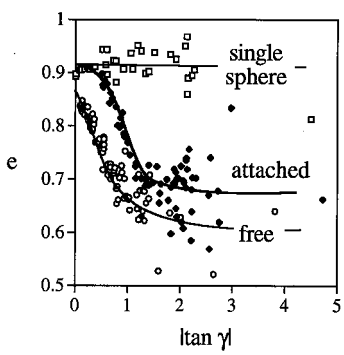

Figure 8 : *Variation of the coefficient of normal restitution with tangent of the angle of incidence for the "single sphere" (squares, Fig. 3), "attached" spheres (diamonds, Fig. 6) and "free" sphere (circles, Fig. 7). The lines are visual fits through the data.*