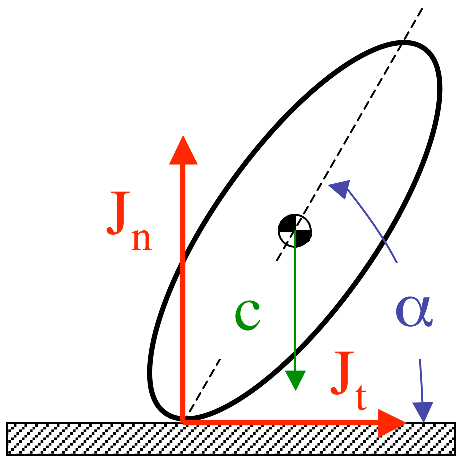
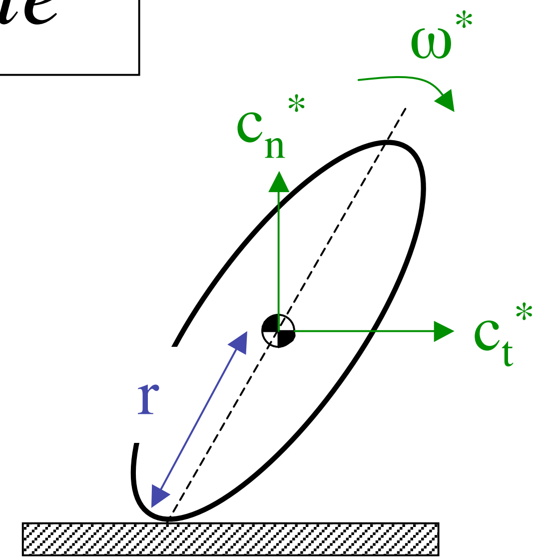
Absurd impact model

Counter-example





Impulse: $\underline{J} = m(\underline{c}^* - \underline{c}) \longrightarrow J_n/m = c_n^* - c$ (1)

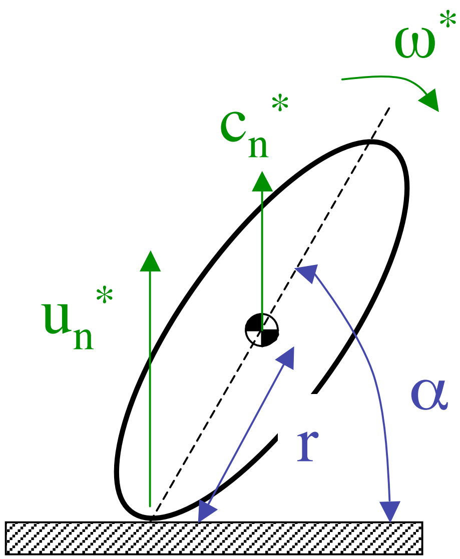
Angular momentum: $I\,\omega^* = J_n\, r \cos\alpha - J_t\, r \sin\alpha$ (2)

Coulomb friction: $J_t = \mu\, J_n \longrightarrow I\,\omega^* = J_n\, r \cos\alpha\,(1 - \mu \tan\alpha)$

$\mu\, J_n/m = c_t^*$

c_n^*, c_t^* and ω^* all derive from J_n

Normal kinematic restitution (Newton)



$$u_n^* = c_n^* + \omega^* r \cos\alpha = -e c$$

1, 2 →

$$J_n = -\frac{m(1+e)c}{1+\eta(1-\mu\tan\alpha)}$$

$\eta = m r^2 \cos^2\alpha / I$ and $\eta^\dagger = \eta\,(1 - \mu \tan\alpha)$

$$\Delta E_{rot} = \frac{I}{2}\omega^{*2} = \frac{m}{2}c^2\frac{(1+e)^2\eta^{\dagger 2}}{\eta(1+\eta^\dagger)^2}$$ >0

normal tangential initial

$$\Delta E_{trans} = \frac{m}{2}c^2\left[\left(\frac{\eta^\dagger - e}{1+\eta^\dagger}\right)^2 + \frac{\mu^2(1+e)^2}{(1+\eta^\dagger)^2} - 1\right]$$ > 0 or < 0

Possible violations

$$\frac{\Delta E_{tot}}{\frac{m}{2}c^2} = \frac{(1+e)}{(1+\eta^\dagger)^2}[\eta(1-\mu\tan\alpha)^2(1+e)-(1-e)$$
$$-2\eta(1-\mu\tan\alpha)+\mu^2(1+e)]$$

$\eta = m\, r^2 \cos^2\alpha / I$ and $\eta^\dagger = \eta\,(1 - \mu \tan\alpha)$

$$\textit{If } \mu = 0,\ \Delta E_{tot} = \frac{m}{2}c^2\left(\frac{e^2-1}{1+\eta}\right) < 0$$

But, if for example $\eta^\dagger \approx 0$, $\Delta E_{tot} \approx \frac{m}{2}c^2(1+e)\left[\mu^2(1+e)-(1-e)\right]$,

energy dissipation is violated when $\mu > \sqrt{\frac{1-e}{1+e}}$.

with $e = 0.9$, violation for $\mu > 0.23$.