Flow Development of a Sheared Collisional Granular Flow

Haitao Xu and Michel Y. Louge
Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, USA

James T. Jenkins
Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, NY 14853, USA

ABSTRACT: We analyze the development of a collisional granular flow without gravitational accelerations in a shear cell shaped as a racetrack. Variations of the cross-sectional averaged solid volume fraction, mean velocity and fluctuation energy along the straight region of the cell are captured by an integral technique similar to the treatment of boundary layers. The results compare well with data from molecular dynamical simulations of the cell. The theory captures the role of side walls and streamwise temperature variations on the flow development.

INTRODUCTION

The segregation of flowing collisional granular materials has been the object of recent theories. Jenkins & Mancini (1989) derived explicit expressions for the constitutive relations of a flowing binary mixture of slightly inelastic spheres. Arnarson & Jenkins (1999) simplified the theory by expressing the transport coefficients in terms of first order perturbations in the masses and radii difference. Arnarson et al. (1999) applied the simplified theory to a fully developed rectilinear flow.

Louge et al. (2000) attempted to test these theories in microgravity using a shear cell shaped as a racetrack. They hoped that the volume fraction, velocity and fluctuation energy of the two species of spheres would become fully developed, i.e. independent of streamwise direction, somewhere in the straight section of the cell. Unfortunately, their numerical simulations revealed that the curved regions of the cell affected the flow too far into the straight section for fully developed conditions to be achieved in an apparatus of reasonable dimensions.

To elucidate flow development in the straight sections, we focus here on flows of monodisperse spheres. We derive balance equations for variables averaged through the channel cross-section in a manner analogous to the integral treatment of a boundary layer. We then compare results with numerical simulations.

THEORY

The conservation laws for a collisional granular flow differ from those of an ordinary fluid by the presence of fluctuation energy dissipation. In a steady flow without gravitational accelerations, they are:

$$\nabla \cdot (\rho \mathbf{u}) = 0 \qquad (1)$$

$$\nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla\left[p - \kappa(\nabla \cdot \mathbf{u})\right] + \nabla \cdot \boldsymbol{\tau} \qquad (2)$$

$$\nabla \cdot \left[(3/2)\rho \mathbf{u} T\right] = -\nabla \cdot \mathbf{q} - \gamma + \left\{\left[-p + \kappa(\nabla \cdot \mathbf{u})\right]\mathbf{I} + \boldsymbol{\tau}\right\} : \mathbf{S} \qquad (3)$$

where $\rho = \phi\rho_s$ is the bulk density, ϕ is the solid volume fraction, ρ_s is the material density of the grains, $\mathbf{u}$ is their mean velocity, and $\mathbf{I}$ is the identity tensor. The shear stress tensor is $\boldsymbol{\tau} = 2\mu\hat{\mathbf{S}}$, where $\mathbf{S} = (1/2)\left[(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\right]$ is the symmetric part of the velocity gradient and $\hat{\mathbf{S}}$ is the deviatoric part of $\mathbf{S}$. The heat flux is $\mathbf{q} = -k\nabla T$ and p, κ, μ, k, and γ are the pressure, bulk viscosity, shear viscosity, heat conductivity and energy dissipation rate, respectively. Finally, the granular temperature is $T \equiv \langle C^2 \rangle / 3$, where C is the fluctuation velocity.

Jenkins & Richman (1985) derived the following constitutive relations from the kinetic theory,

$$p = \rho T(1+4G) \tag{4}$$

$$\kappa = \left(8/3\sqrt{\pi}\right)\rho\sigma T^{1/2}G \tag{5}$$

$$\mu = \left(8J/5\sqrt{\pi}\right)\rho\sigma T^{1/2}G \tag{6}$$

$$k = \left(4M/\sqrt{\pi}\right)\rho\sigma T^{1/2}G \tag{7}$$

$$\gamma = \left(24/\sigma\sqrt{\pi}\right)\left(1-e_{eff}\right)\rho T^{3/2}G \tag{8}$$

where σ is the diameter of grains, and G, J, and M are known functions of ϕ. The effective coefficient of normal restitution e_{eff} captures frictional energy losses in the flow (Zhang 1993).

We analyze the flow development in the straight section of the cell sketched in Figure 1. The channel is bound by an inner moving wall of speed U to which cylindrical bumps are affixed (Louge et al. 2000), by a stationary bumpy outer wall, and by two flat side walls. Along the straight section of length L, the x-axis is in the flow direction, the y-axis is perpendicular to the bumpy boundaries and the z-axis is normal to the side walls.

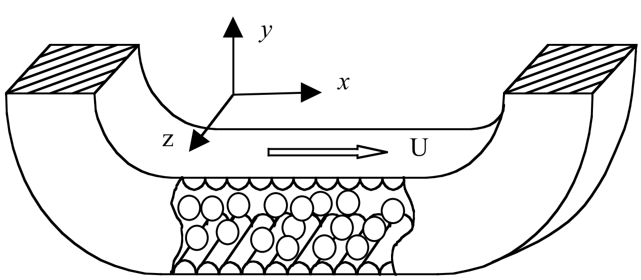


Figure 1. Sketch of the bottom half of the shear cell.

Our analysis focuses on streamwise variations of volume fraction, velocity and temperature averaged through the channel cross-section. We assume that the velocity component in z-direction vanishes, i.e. $\mathbf{u}=(u, v, 0)$. Then, the average velocity is

$$\bar{u} \equiv \frac{1}{HW}\int_{-W/2}^{W/2}\int_{0}^{H} u\,dy\,dz, \tag{9}$$

where H and W are shear cell height and width, respectively.

We further assume that ϕ and T vary along x only, and that u and v are independent of z. However, to capture the effects of side walls, we retain the heat flux and shear stress on surfaces normal to z. Because $v \ll u$ and $\partial/\partial y \gg \partial/\partial x$, $\partial v/\partial x$ is negligible compared with other terms, but we keep $\partial v/\partial y$ and evaluate it using mass conservation.

To make the integration in Eq. (9) tractable, we assume that the transverse profile of streamwise velocity at an arbitrary cross-section is parallel to its fully developed counterpart. In this case, the integration of the momentum balance requires that the velocity profile have the form

$$\begin{aligned} u(x,y) = \bar{u}(x) - \left[u_1(x) - u_0(x)\right]/2 + A/12 \\ + \left[u_1 - u_0 - A/2\right](y/H) + (A/2)(y/H)^2 \end{aligned} \tag{10}$$

where u_1 and u_0 are velocities at the top and bottom bumpy boundaries, respectively; A is obtained by integrating the momentum equation in the fully developed region,

$$A = \frac{2\mu_{sw}}{W}\frac{p}{\mu} = \frac{5\sqrt{\pi}(1+4G)}{4GJ}\frac{\mu_{sw}}{\sigma W}T_{FD}^{1/2} \tag{11}$$

where μ_{sw} is the frictional coefficient of the side walls, T_{FD} is the temperature in the fully developed region. For a given geometry, A depends only on ϕ and T_{FD} and it is determined from the numerical solution of the fully developed state. In this formulation, the quantities $u_{sh}=u_1(x)-u_0(x)$ and A are thus constants.

We then integrate the momentum and energy equations in y- and z-direction to find the evolution of $\bar{u}$, ϕ and T along x. For simplicity, we assume that the average along z of the square of the shear stress τ_{xy} is equal to the square of its average, and that the shear stress τ_{xz} is proportional to z and vanishes at the centerline by symmetry. The shear stresses and heat fluxes at all boundaries are retained in the conservation equations. At the flat side walls, we use the boundary conditions derived by Jenkins (1992) and Jenkins & Louge (1997) to evaluate the shear stress and heat flux at the wall, respectively; at the bumpy boundaries, we use the nonlinear boundary conditions derived by Jenkins et al. (2000). These boundary conditions have the form:

$$\tau_w / p = f_\tau(U-u, T, \phi, \mu_w) \tag{12}$$

$$q_w / \left(pT^{1/2}\right) = f_q(U-u, T, \phi, \mu_w, e_w) \tag{13}$$

where τ_w and q_w are, respectively, the shear stress and heat flux at the boundary; U-u is the slip velocity there, and e_w and μ_w are the coefficient of restitution and friction of the wall, respectively. Equations (12) and (13) capture geometrical details of the bumpy boundaries and flat side walls.

After evaluating the integrals and using the boundary conditions, we obtain

$$Q = \phi\bar{u} \tag{14}$$

where Q is a constant and

$$\frac{d}{dx}\left(\frac{KQ}{\phi^2}\frac{d\phi}{dx}\right)+\rho_s\left(\frac{u_{sh}^2}{12}-\frac{Q^2}{\phi^2}\right)\frac{d\phi}{dx} + \frac{dp}{dx}-\frac{\tau_{y1}-\tau_{y0}}{H}+\frac{2\tau_{sw}}{W}=0, \tag{15}$$

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)-\frac{3}{2}\rho\bar{u}\frac{dT}{dx}+\frac{q_{y1}+q_{y0}}{H}+\frac{2q_{sw}}{W} + \frac{\mu}{H^2}\left(u_{sh}^2+\frac{A^2}{12}\right)+\frac{\tau_{sw}^2}{3\mu}-\frac{24}{\sqrt{\pi}}\left(1-e_{eff}\right)\frac{\rho T^{3/2}}{\sigma}G - p\frac{d\bar{u}}{dx}+K\left[1+\frac{1}{\bar{u}^2}\left(\frac{u_{sh}^2}{12}+\frac{A^2}{720}\right)\right]\left(\frac{d\bar{u}}{dx}\right)^2=0, \tag{16}$$

where $K \equiv \kappa + 4\mu/3$. In these Eqs, the shear stresses τ_{y1} and τ_{y0} at the top and bottom bumpy boundary, the shear stress τ_{sw} at the flat side wall, and the heat fluxes through the top and bottom bumpy boundaries and the flat side walls q_{y1}, q_{y0} and q_{sw} are given in terms of T, ϕ and impact parameters by Eqs (12) and (13). In these one-dimensional equations, we further employ the mass conservation equation (14) and the equation of state (4) to eliminate $\bar{u}$ and p, respectively, in terms of volume fraction and temperature.

The resulting momentum and energy equations are two ordinary differential equations for the evolution of solid volume fraction and temperature. They are coupled through transport coefficients and are solved numerically subject to known boundary conditions at the inlet and outlet of the straight section.

RESULTS AND DISCUSSION

As Figure 2 shows, predictions of the one-dimensional theory agree with data from the computer simulations. Figure 3 shows the relative magnitude of each term in the energy balance, except heat conduction and convection along the flow direction, which are small. Clearly, the working of normal stresses is important near the exit where the flow undergoes rapid compression. The resulting rise in the granular temperature then permits the high volume fraction at the edge of the curved region to propagate upstream. To illustrate this, Figure 2 shows the predicted profiles when the working of normal stresses is ignored.

To evaluate the role of side walls, Figure 4 contrasts the profiles from both theory and simulation in channels with smooth and frictional side walls. Because frictional side walls reduce the mean velocity, the high volume fraction at the edge of the curved region can propagate farther upstream, thus shrinking the fully-developed region and moving it upstream.

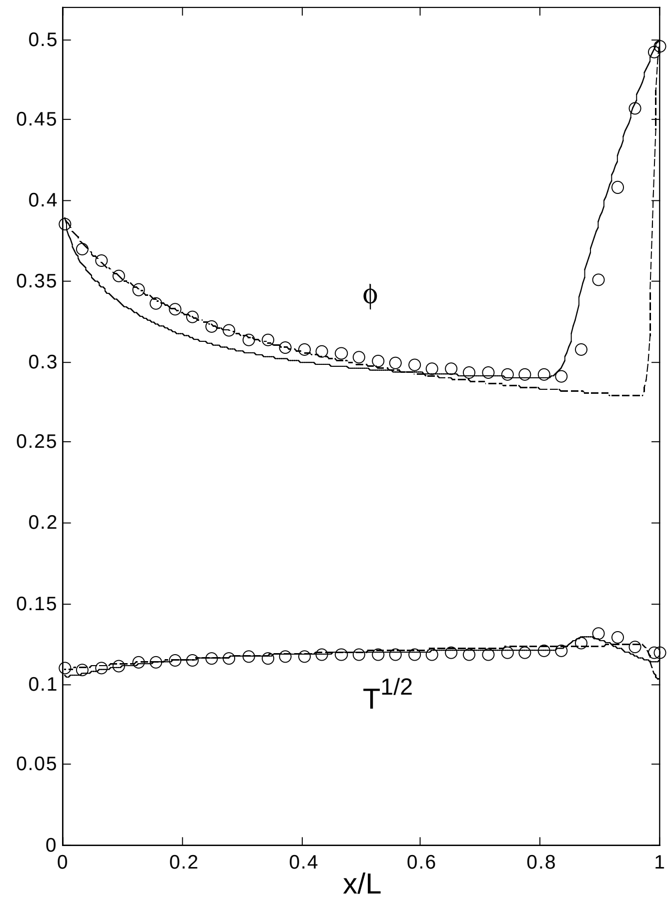
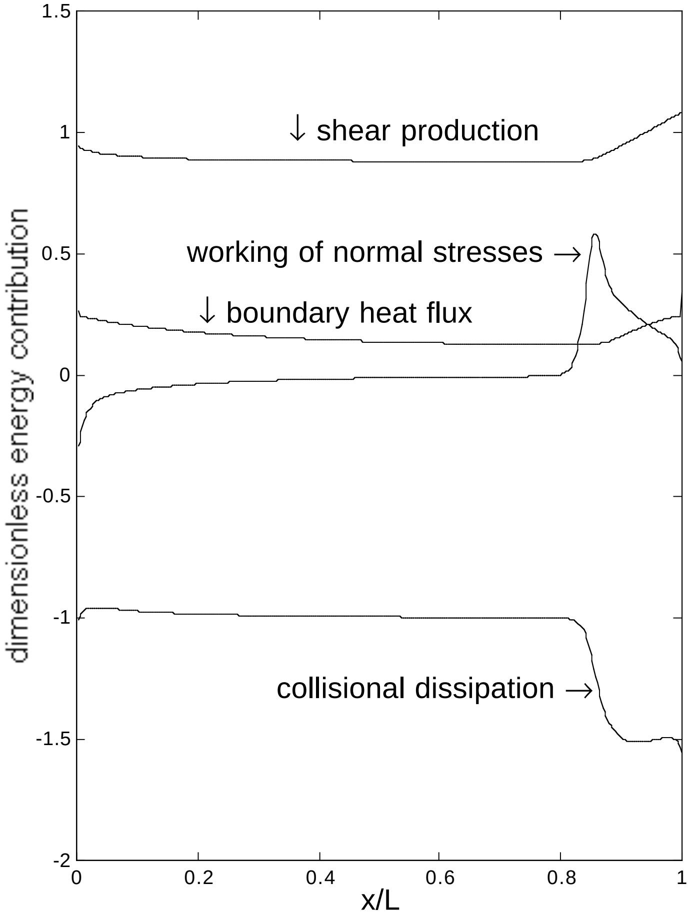


Figure 2. Comparison of the development theory (lines) and molecular dynamical simulations (symbols) for a shearing cell with L/σ=140, W/σ=4 and H/σ=6. Impact parameters (normal restitution, friction, tangential restitution) are (0.95, 0.1, 0.4) for binary impacts, (0.95, 0,0.4) for impacts of the spheres with smooth flat walls and (0.85, 0.1, 0.4) for impacts with boundary bumps. The solid lines indicate the complete theory; the dashed lines represent an incomplete theory where the working of normal stresses is ignored. The top and bottom curves are volume fraction and fluctuation velocity profiles, respectively.



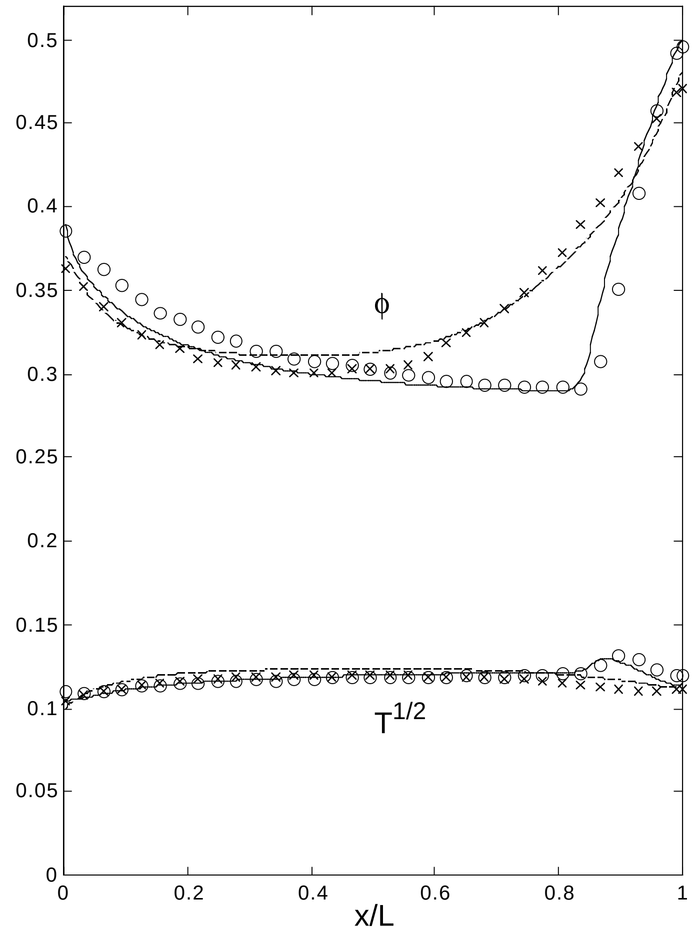


Figure 3. Contributions to the energy balance along the flow in Figure 2, normalized by γ_{FD}, the collisional dissipation rate in the fully developed region. On the left, from top to bottom: production by the working of the mean shear, fluctuation energy input from the boundary, working of the normal stress, and collisional energy dissipation.

Figure 4. Effects of flat side walls on flow development. See Figure 2 for conditions and symbols. The solid lines represent a theory with smooth side walls; the dashed lines are for side walls with friction coefficients μ_{sw}=0.1.

REFERENCE

Arnarson, B.O. & J.T. Jenkins 1999. Binary mixtures of inelastic spheres: simplified constitutive theory, under review.

Arnarson, B.O. et al. 1999. Binary mixtures of inelastic spheres: a boundary-value problem, under review.

Jenkins, J.T. & M.Y. Louge 1997. On the flux of fluctuation energy in a collisional grain flow at a flat frictional wall,. *Phys. Fluids* 9:2835-2840.

Jenkins, J.T. & F. Mancini 1989. Kinetic theory for binary mixtures of smooth, nearly elastic spheres. *Phys. Fluids A*. 1:2050-2059.

Jenkins, J.T. & M.W. Richman 1985. Grad's 13-moment system for a dense gas of inelastic spheres. *Arch. Rat. Mech. Anal*. 87:355-377.

Jenkins, J.T. 1992. Boundary conditions for rapid granular flows: flat frictional walls. *J. Applied Mech*. 59:120-127.

Jenkins, J.T et al. 2000. Nonlinear boundary conditions for collisional grain flows, in preparation.

Louge, M.Y. et al. 2000. Granular segregation in collisional shearing flows. in *Mechanics for a New Millenium*, A. Aref & J.W. Phillips eds, Kluver Academic Publishers, in press.

Zhang, C. 1993. Kinetic theory for rapid granular flows. Ph.D Thesis. Cornell University.