# *Role of pore pressure gradients in geophysical flows over permeable substrates*

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Aussois, May 24, 2013

Shields, A. (1936). Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung, Mitteilungen der Preußischen Versuchsanstalt für Wasserbau 26. Berlin: Preußische Versuchsanstalt für Wasserbau.

## *Shields erosion*

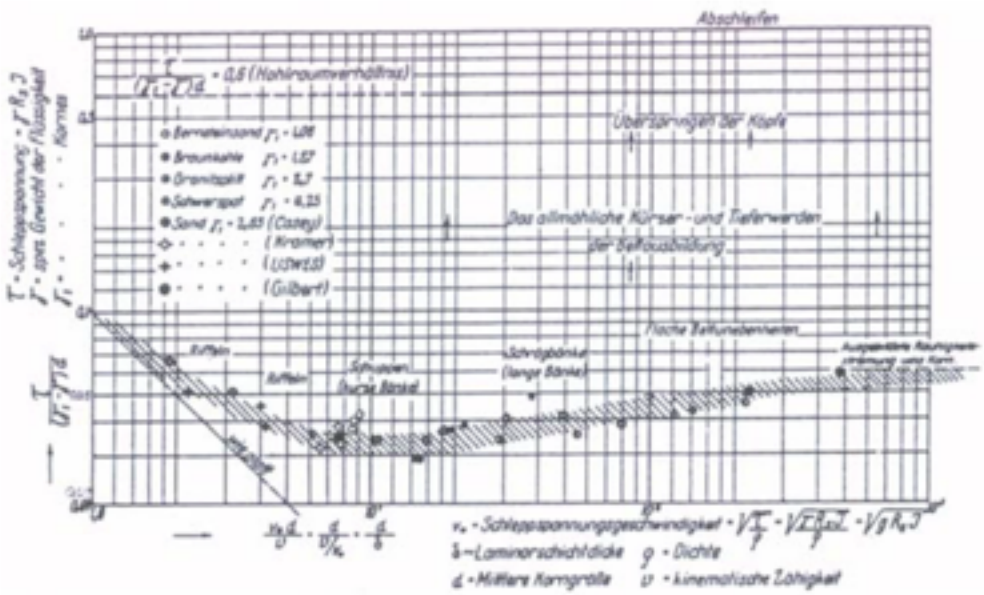


Abb. 6.

Schleppspannungsbeiwert $\frac{\tau}{(\gamma_1-\gamma)\,d}$ gegen die Reynold'sche Zahl des Kornes $\frac{v_* d}{\nu}$.

Shields, A. (1936). Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung, Mitteilungen der Preußischen Versuchsanstalt für Wasserbau 26. Berlin: Preußische Versuchsanstalt für Wasserbau.

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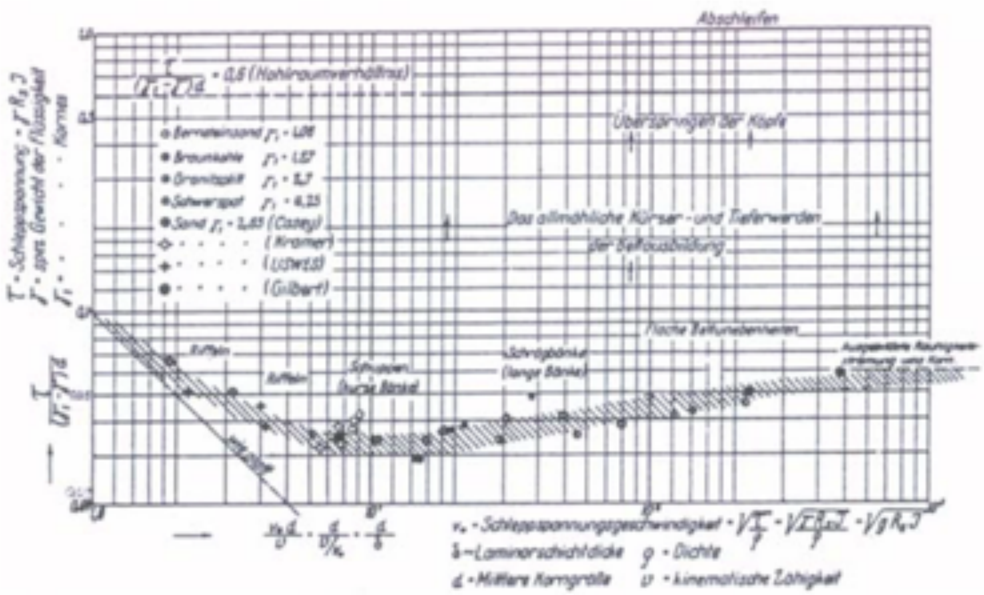

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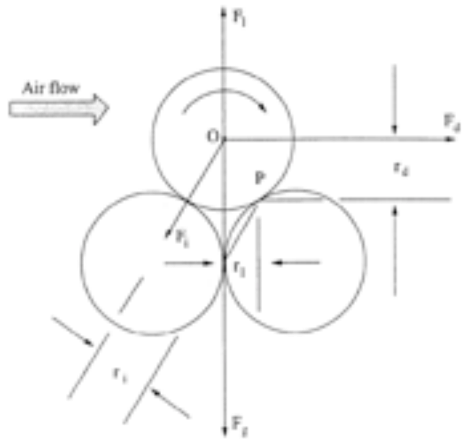



**Figure 1.** Forces acting on a particle resting on the surface under the influence of an airstream, including the aerodynamic drag $F_d$, the aerodynamic lift $F_l$, the gravity force $F_g$, the moment $F_m$, and the cohesive force $F_i$; $r_d$, $r_l$, $r_m$, and $r_i$ are moment arm lengths associated with $F_d$, $F_l$ and $F_g$, $F_m$, and $F_i$, respectively. O is the center of gravity of the particle, and P is the pivot point for particle entrainment.

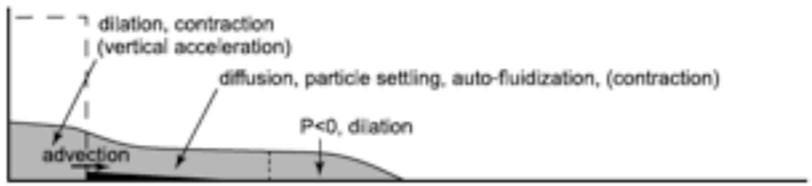
Shao, Y., and H. Lu (2000), A simple expression for wind erosion threshold friction velocity, J. Geophys. Res., 105, 22437-22443.

# *Flow over porous media*

R. M. Iverson, Regulation of landslide motion by dilatancy and pore pressure feedback, J. Geophys. Res. 110, F02015 (2005).

R. M. Iverson, et al, Positive feedback and momentum growth during debris-flow entrainment of wet bed sediment, Nature Geoscience 4, 116-121 (2010).

Roche, O., S. Montserrat, Y. Niño, and A. Tamburrino (2010), Pore fluid pressure and internal kinematics of gravitational laboratory air-particle flows: Insights into the emplacement dynamics of pyroclastic flows, J. Geophys. Res., 115, B09206.




T. Yamamoto, H. L. Koning, H. Sellmeijer, and EP Van Hijum, On the response of a poro-eleastic bed to water waves, J. Fluid Mech. 87, 193-206 (1978).

# *Two examples*

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# Powder snow avalanches


Sovilla, Burlando and Bartelt, JGR (2010)

WSL Institute for Snow and Avalanche Research SLF

# Growth rate

Growth rate of a cloud of height $H'$ and density $\rho'$ due to basal shearing at a shear velocity $u^*$

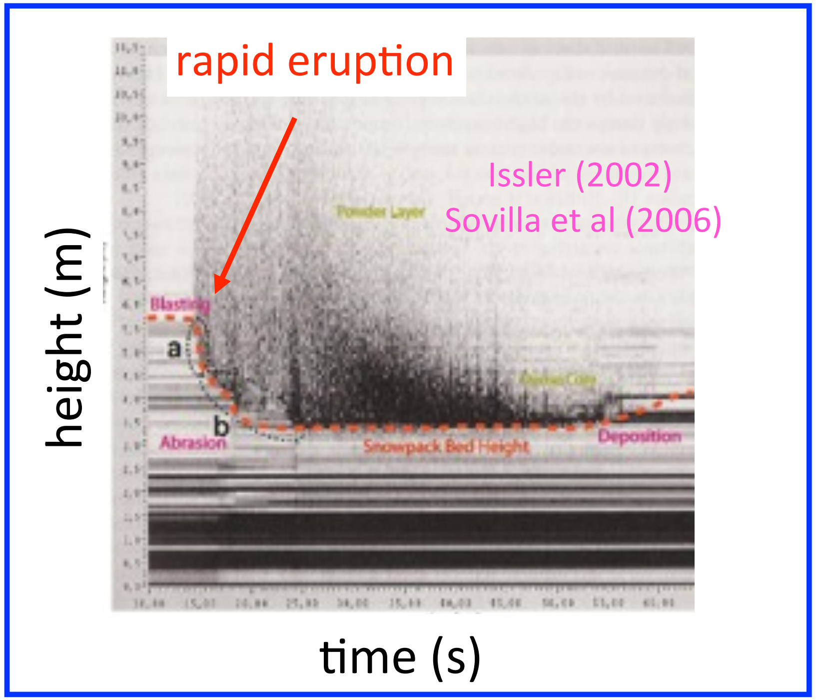
$$\frac{d}{dt}\left[\rho'\left(H'^2 W\right) g H'\right] \sim \rho' u^{*2}\left(H'W\right)u^*$$

rate of potential energy

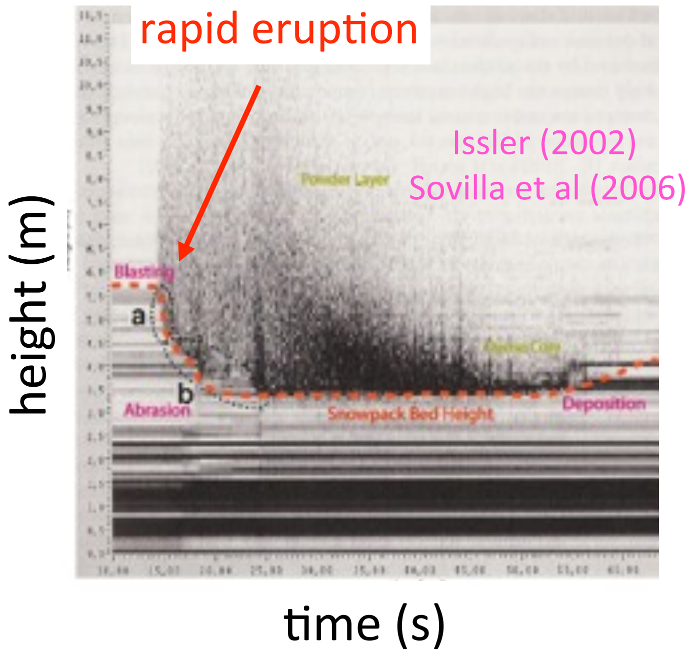
working rate of basal shear stress

$$\frac{dH'}{dt} \sim \frac{u^{*3}}{3gH'} \sim \frac{\left(0.05\times 50\right)^3}{3\times 9.81\times 20} \sim 0.03 \text{ m/s}$$

# *Eruption current*

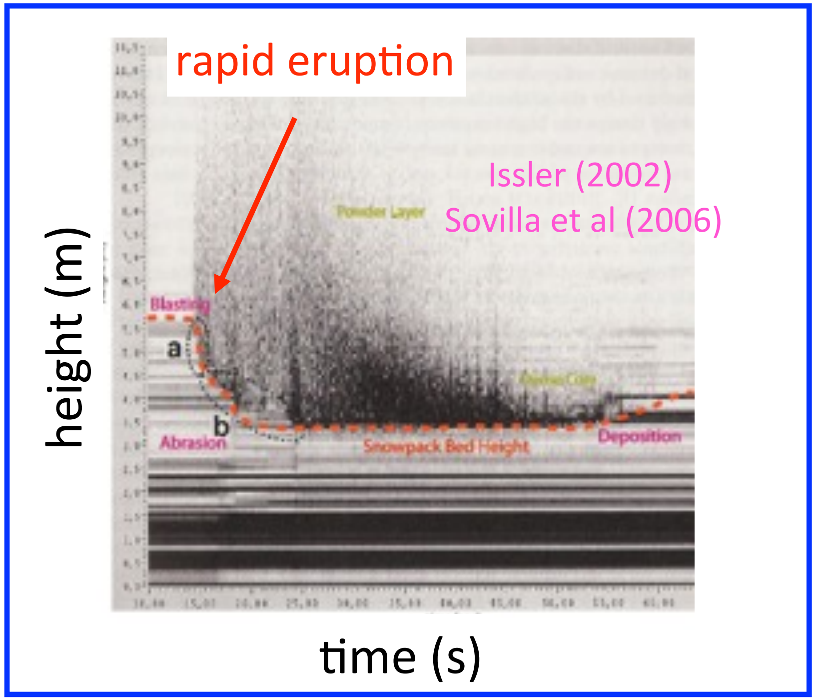


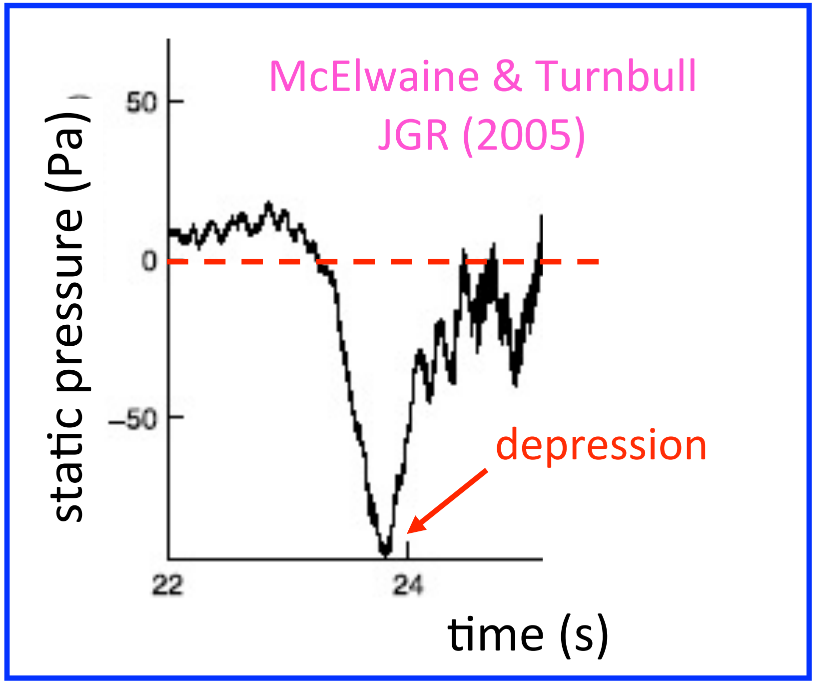
# *Eruption current*




Eruption current =
a gravity current driven by massive frontal eruption

# *Eruption current*




Eruption current =

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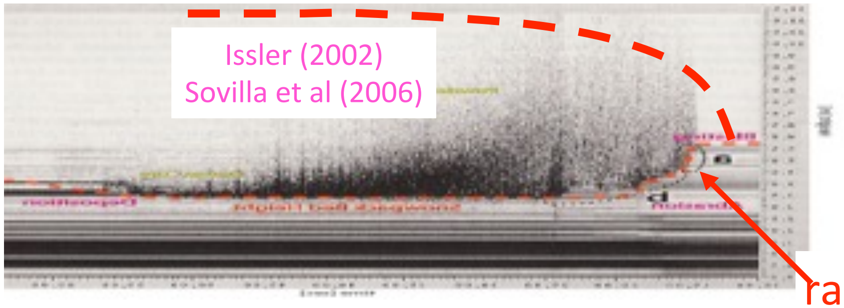
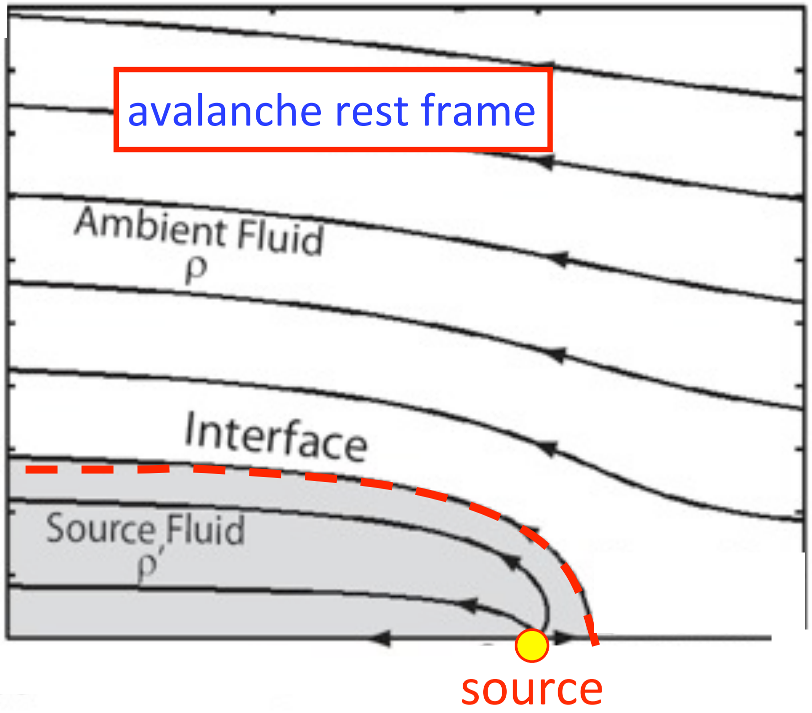
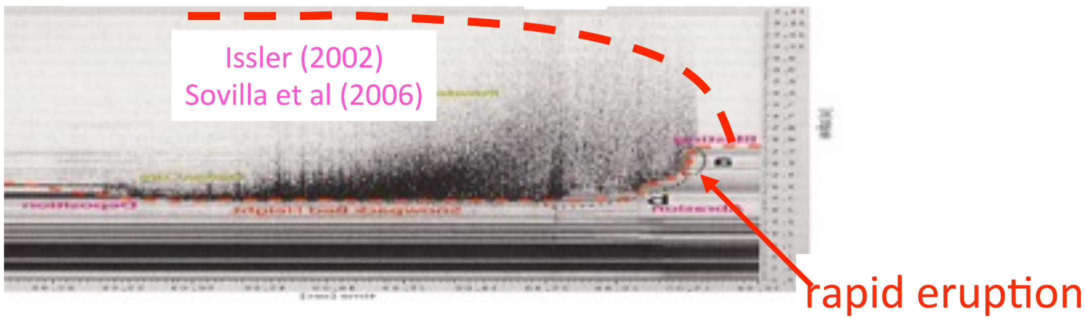
# Eruption current frontal region



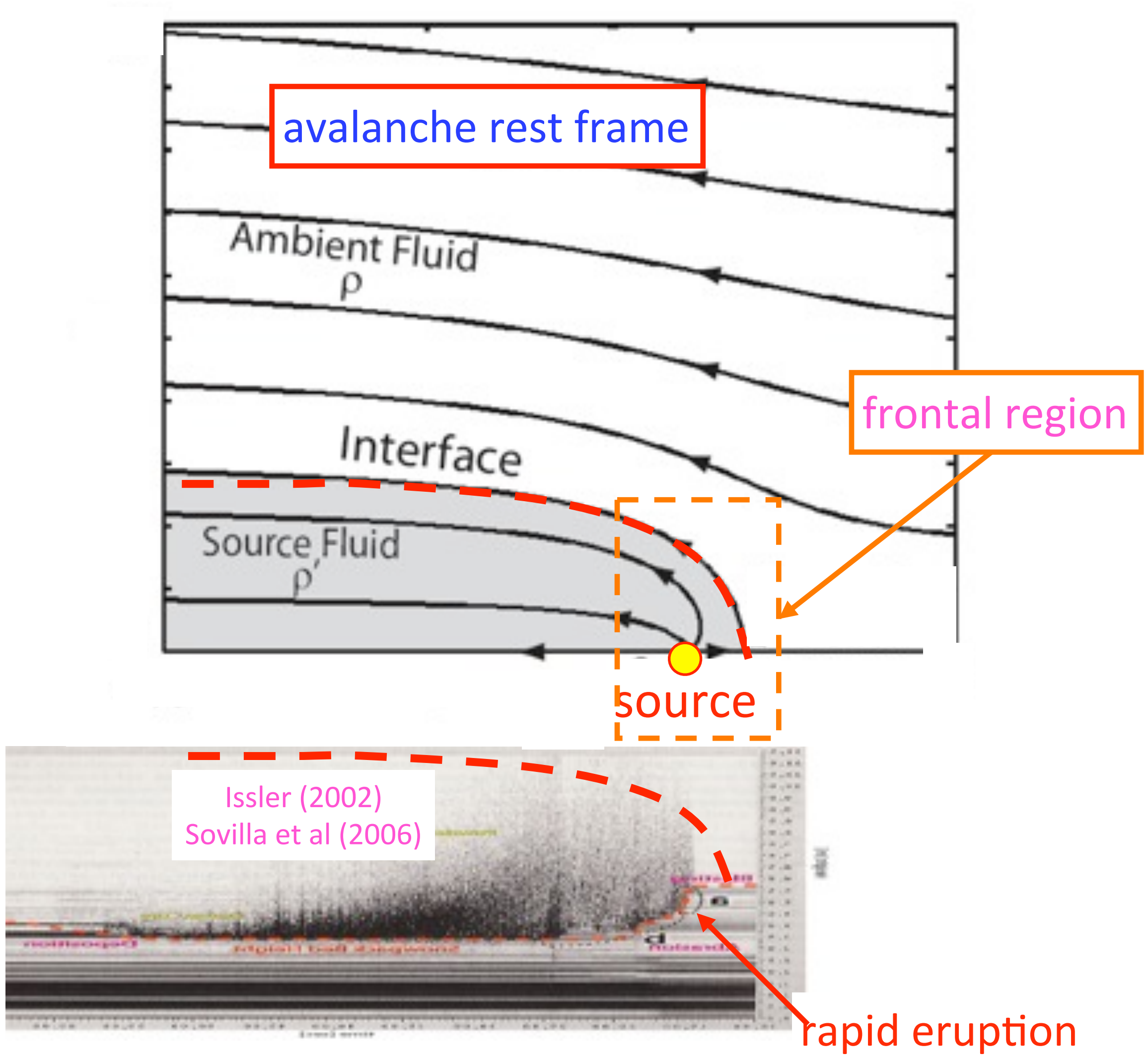

rapid eruption

# Eruption current frontal region

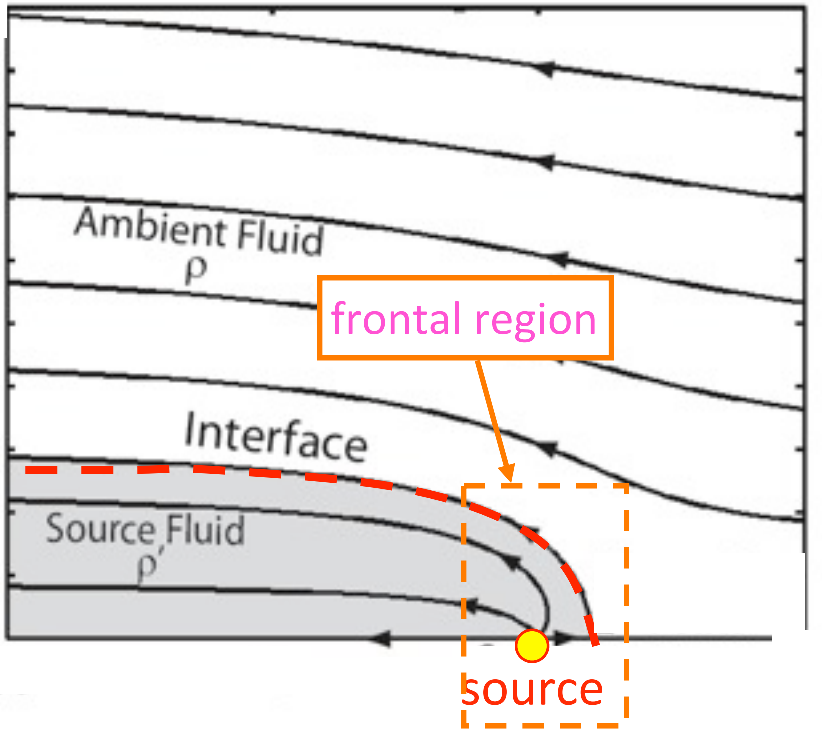


# Eruption current frontal region



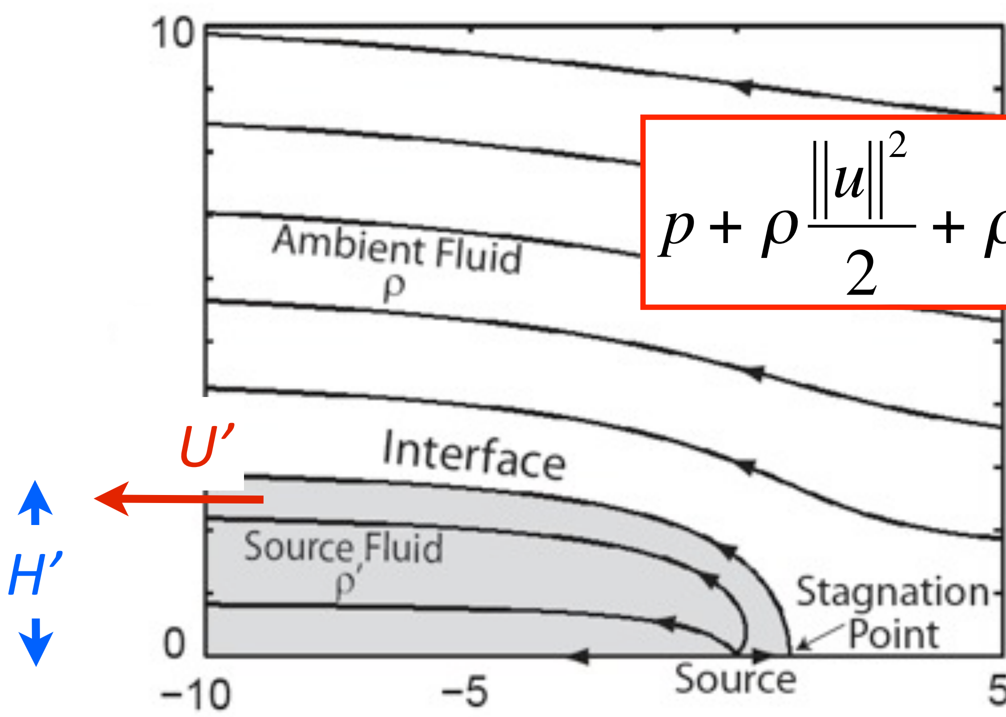
# Principal assumptions




- Negligible basal shear stress
- Negligible air entrainment
- Inviscid
- Uniform mixture density

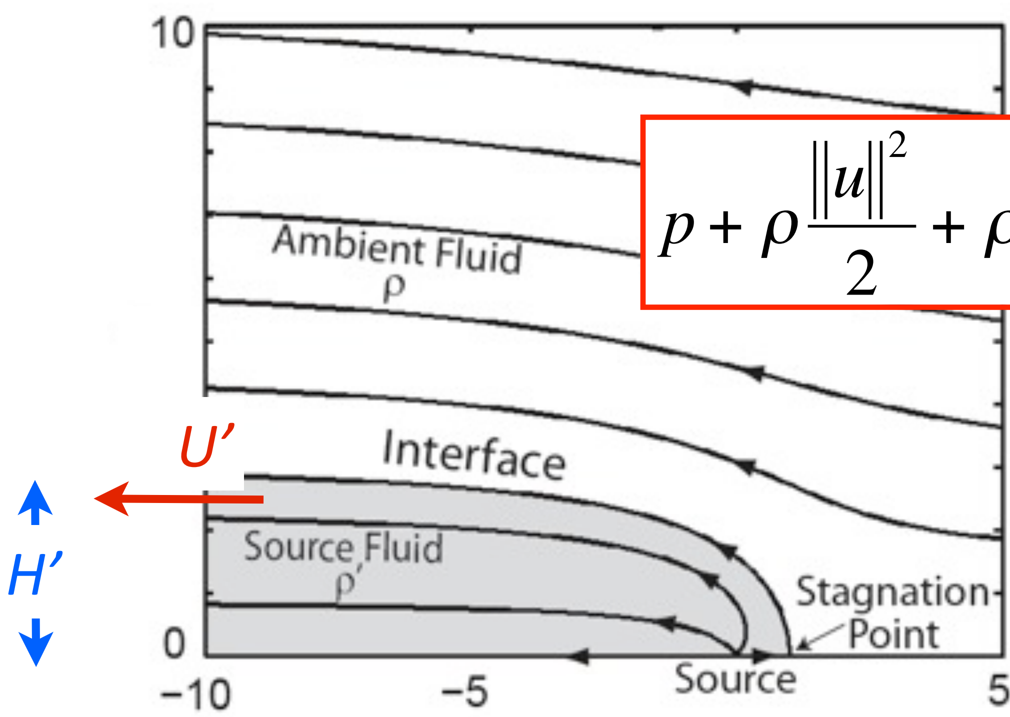
# Potential flow

Rankine, Proc. Roy. Soc. (1864)




$$p + \rho \frac{\|u\|^2}{2} + \rho g z = p' + \rho' \frac{\|u'\|^2}{2} + \rho' g z$$

# Potential flow



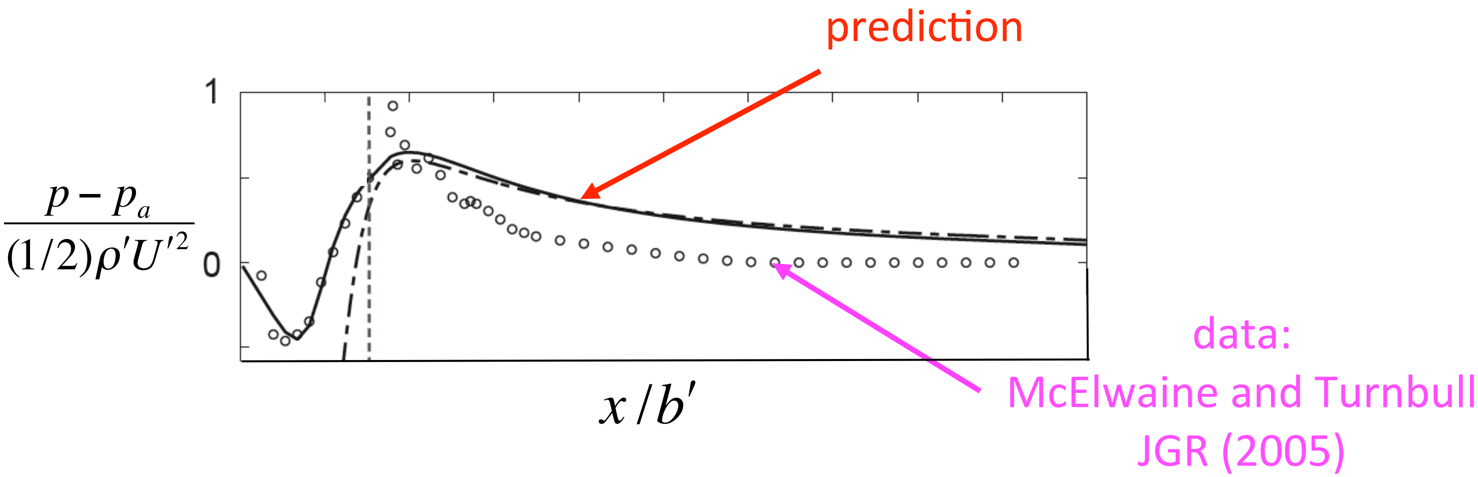

Rankine, Proc. Roy. Soc. (1864)

$$p + \rho \frac{\|u\|^2}{2} + \rho g z = p' + \rho' \frac{\|u'\|^2}{2} + \rho' g z$$

$$\mathrm{Ri} = 2 \frac{(\rho' - \rho)}{\rho'} \frac{gH'}{U'^2}$$

$$\zeta \equiv 1 - \rho / \rho'$$

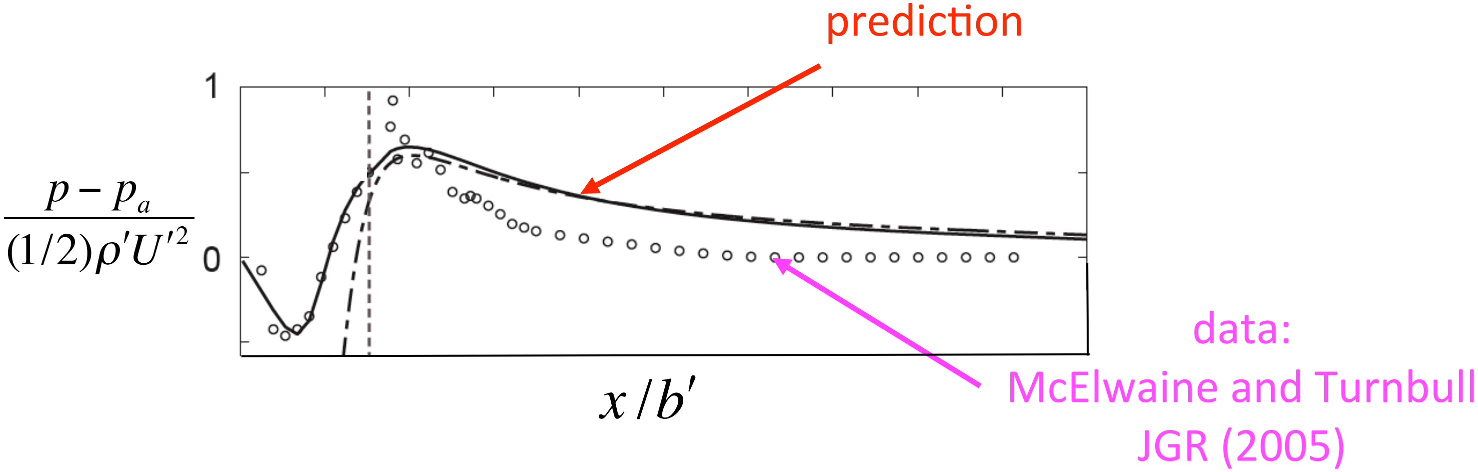
# Static pressure in the cloud




pressure $p$, air density $\rho$, cloud density $\rho'$
stagnation-source distance $b'$
fluidized depth $h'$

$$\frac{p - p_a}{(1/2)\rho' U'^2} = \frac{2(x/b') - 1}{(x/b')^2 + (h'/b')^2}$$

# Static pressure in the cloud




pressure $p$, air density $\rho$, cloud density $\rho'$
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fluidized depth $h'$

$$\frac{p - p_a}{(1/2)\rho' U'^2} = \frac{2(x/b') - 1}{(x/b')^2 + (h'/b')^2}$$

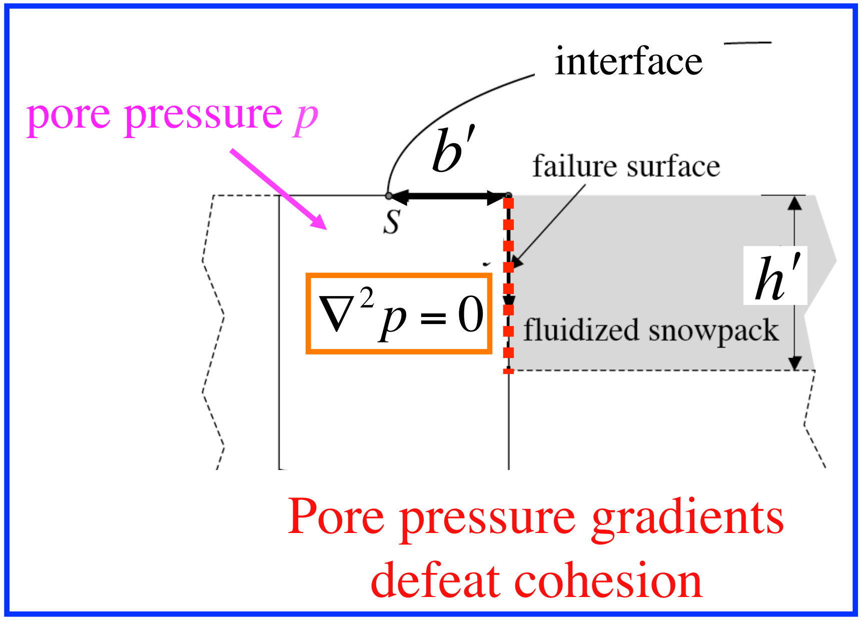
$$\Rightarrow \text{ surface pressure time-history}$$

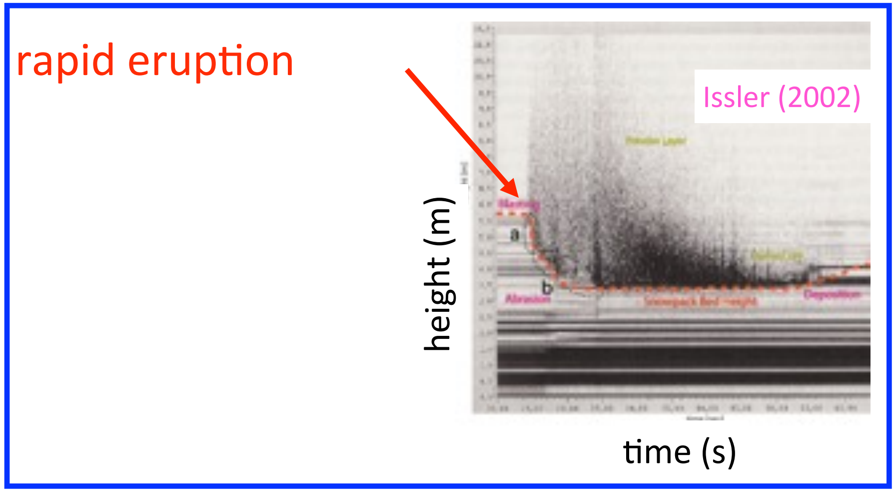
# Porous snow pack

$$u = -\frac{K}{\mu}\nabla p$$

$$\nabla \cdot u = 0$$

$$\nabla^2 p = 0$$



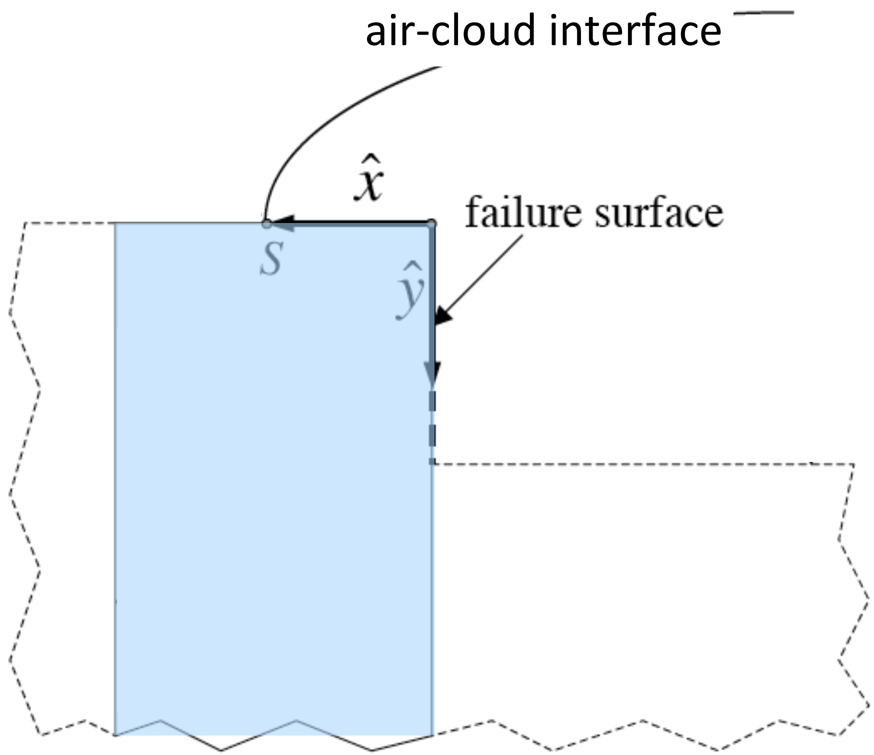



# Force balance in the snowpack

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = -F \cdot \hat{x} = \hat{x} \cdot \nabla p$$

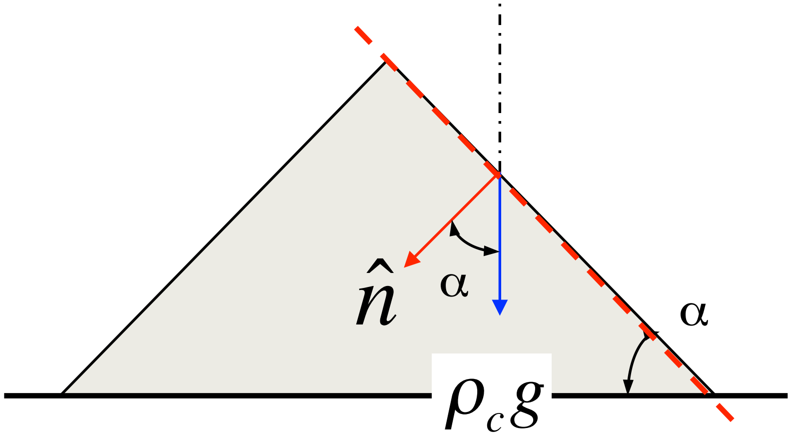
$$\frac{\partial \tau}{\partial x} + \frac{\partial \sigma_y}{\partial y} = -F \cdot \hat{y} - \rho_c g$$

$$= \hat{y} \cdot \nabla p - \rho_c g$$



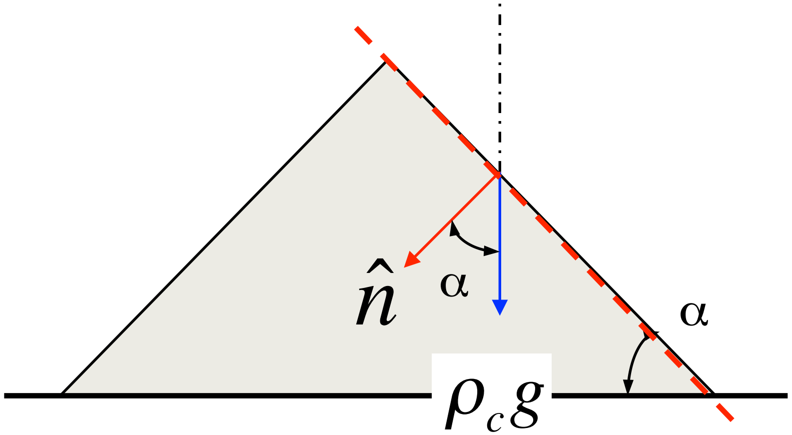

shear stress τ, normal stress σ, gravity g, snowpack density $\rho_c$

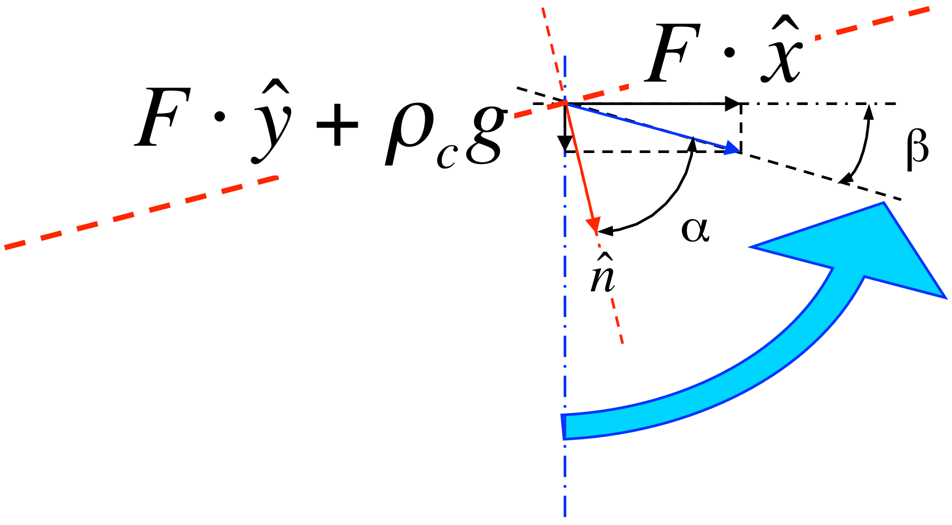
# Snowpack failure




internal friction $\tan\alpha = \mu_E$

# Snowpack failure

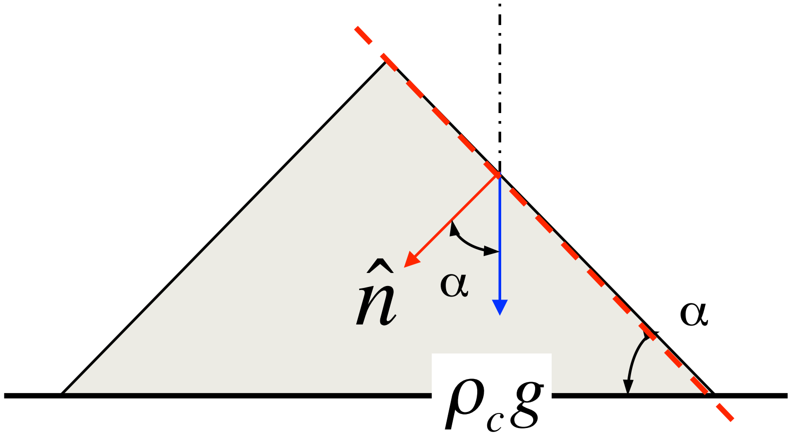


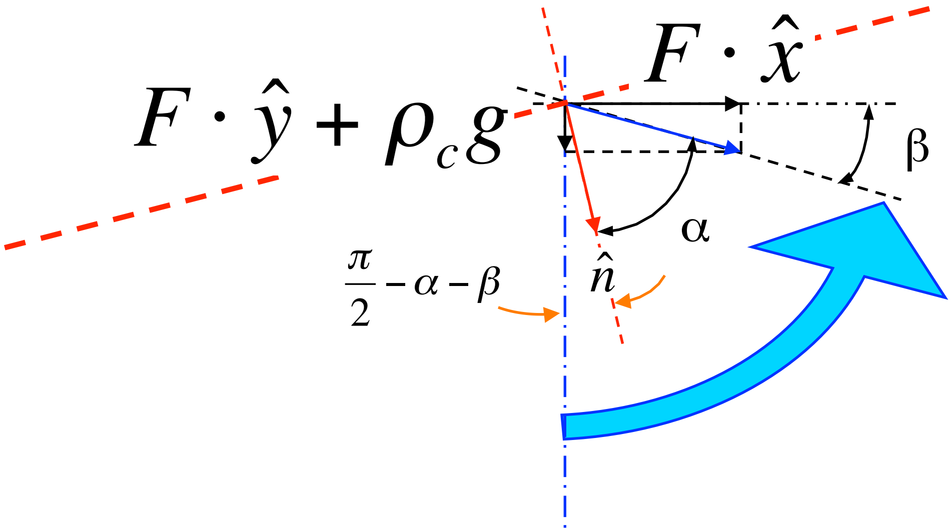




internal friction $\tan\alpha = \mu_E$

$$\tan\beta \equiv \frac{F \cdot \hat{y} + \rho_c g}{F \cdot \hat{x}}$$

# Snowpack failure

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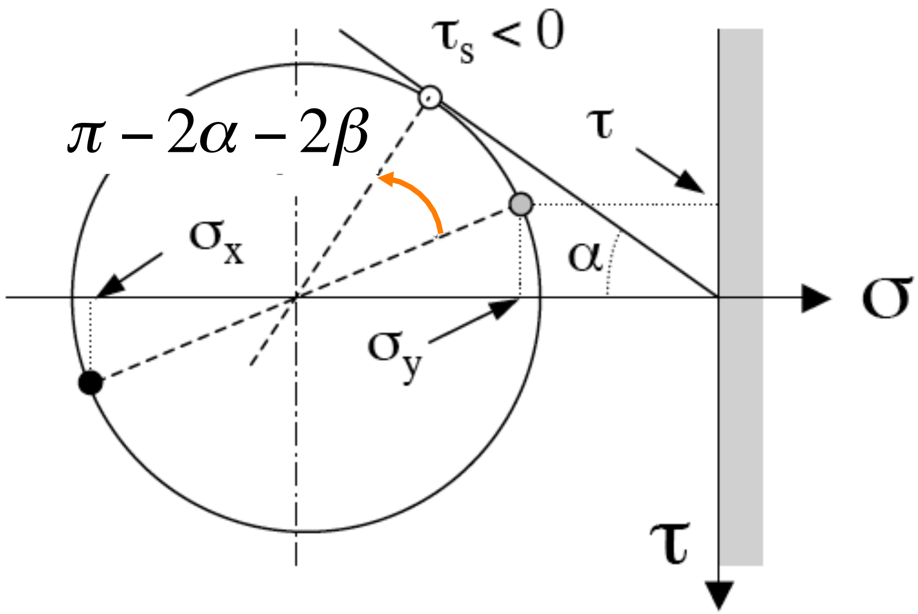
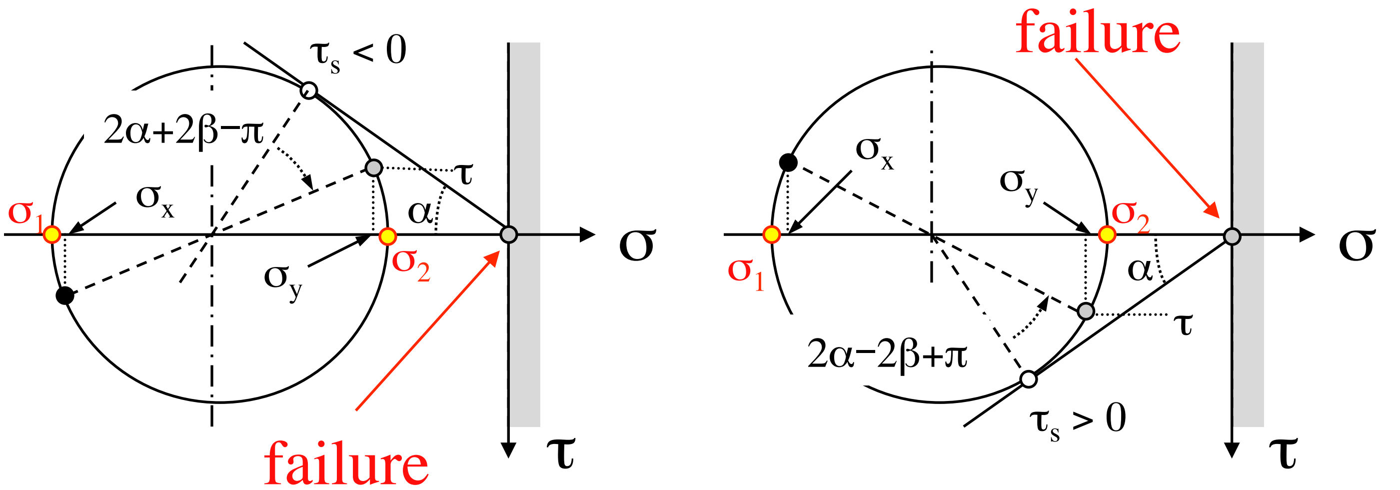


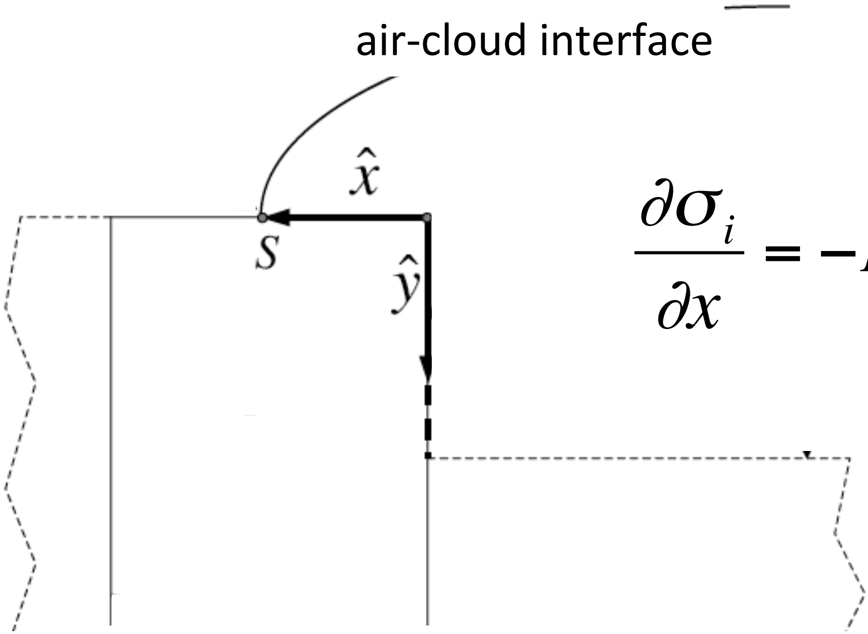
# Relation amongst stresses at fluidization




$$\sigma_x = \sigma_y \left[ \frac{1 - \sin\alpha \sin(3\alpha \mp 2\beta)}{1 + \sin\alpha \sin(3\alpha \mp 2\beta)} \right] \qquad \tau = \pm\sigma_y \left[ \frac{\sin\alpha \cos(3\alpha \mp 2\beta)}{1 + \sin\alpha \sin(3\alpha \mp 2\beta)} \right]$$

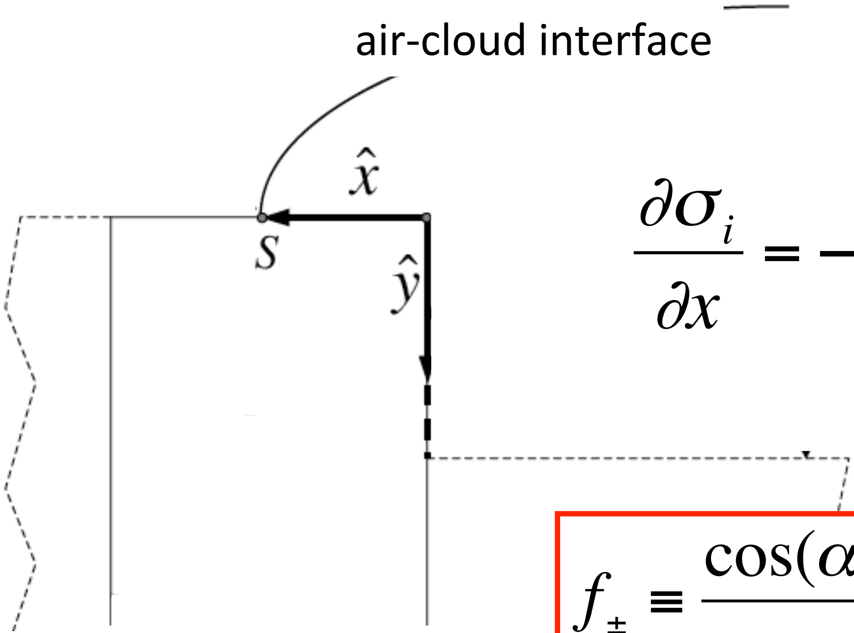
$$\sigma_i = \sigma_y \left[ \frac{1 - (-1)^i \sin\alpha}{1 + \sin\alpha \sin(3\alpha \mp 2\beta)} \right]$$

# Principal stress variations ⊥ failure surface




$$\frac{\partial \sigma_i}{\partial x} = -F \cdot \hat{x}\left[\frac{1-(-1)^i \sin\alpha}{1-\sin^2\alpha}\right] f_{\pm}$$

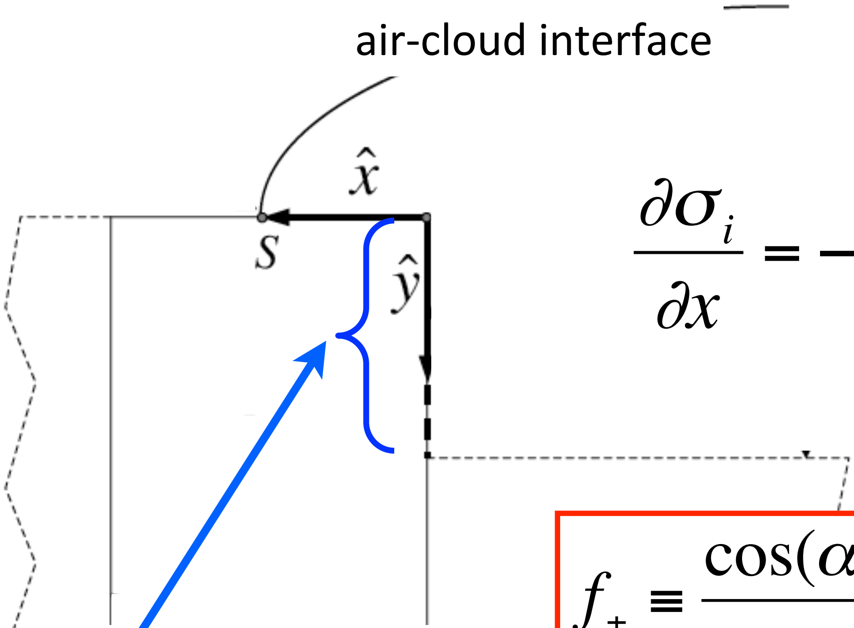
# Principal stress variations ⊥ failure surface




$$\frac{\partial \sigma_i}{\partial x} = -F \cdot \hat{x}\left[\frac{1-(-1)^i \sin\alpha}{1-\sin^2\alpha}\right] f_\pm$$

$$f_\pm \equiv \frac{\cos(\alpha \mp \beta)}{\cos\beta}\left\{\cos\alpha + 2\sin\alpha\sin\left[2(\alpha \mp \beta)\right]\right\}$$

# Principal stress variations ⊥ failure surface

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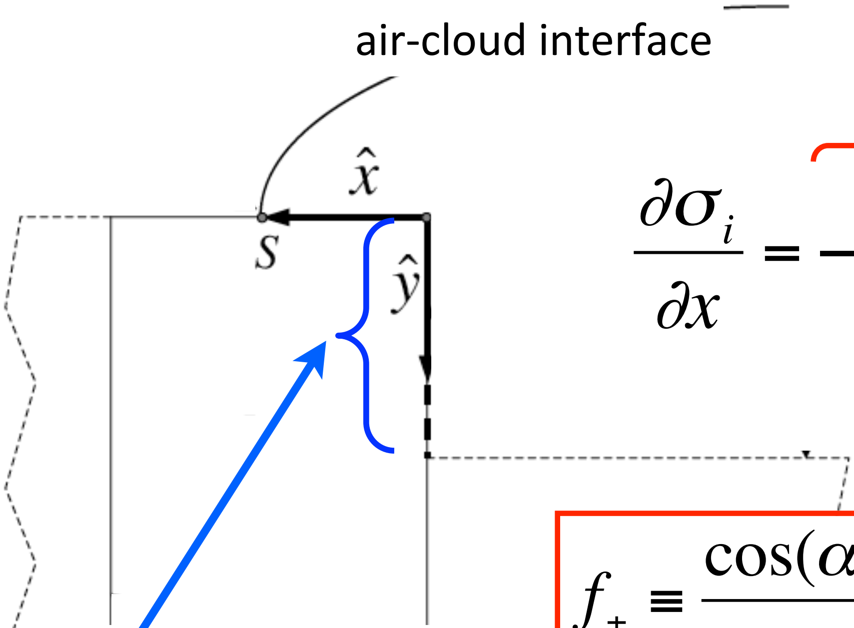
sustainable failure: $\frac{\partial \sigma_i}{\partial x} > 0$

# Principal stress variations ⊥ failure surface



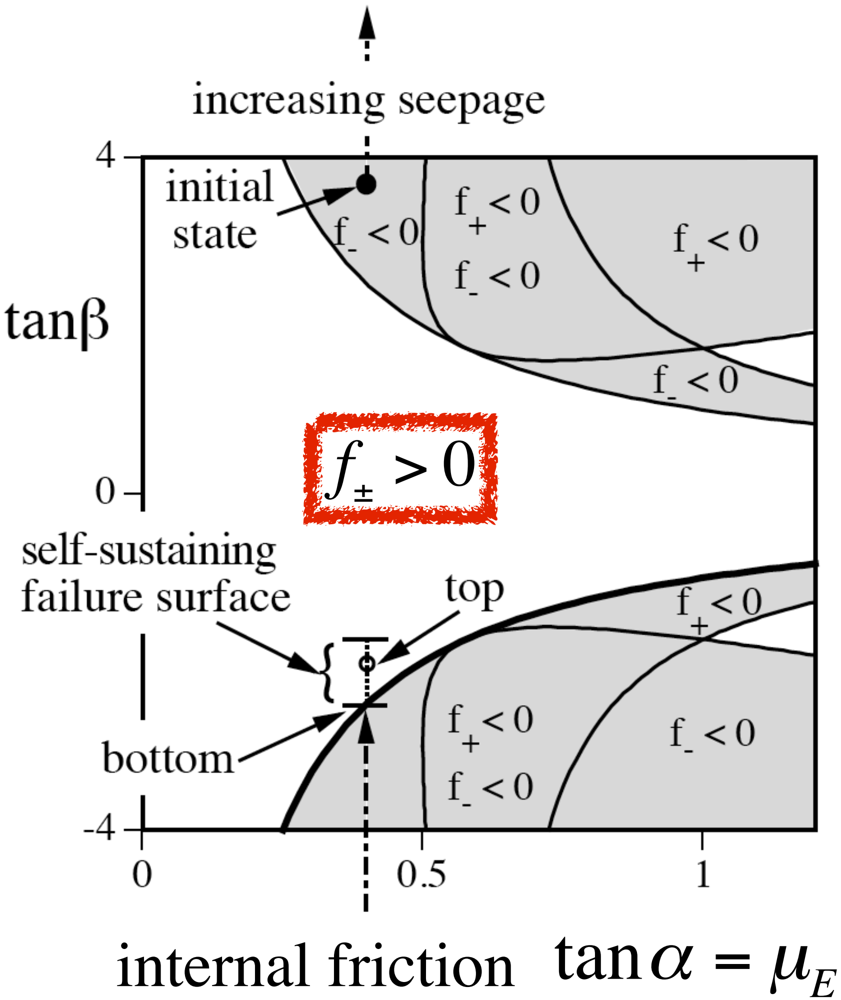

sign of $f_{\pm}$?

$$\frac{\partial \sigma_i}{\partial x} = \underbrace{-F \cdot \hat{x}}_{>0} \underbrace{\left[\frac{1-(-1)^i \sin\alpha}{1-\sin^2\alpha}\right]}_{>0} f_{\pm}$$

$$f_{\pm} \equiv \frac{\cos(\alpha \mp \beta)}{\cos\beta}\left\{\cos\alpha + 2\sin\alpha\sin\left[2(\alpha \mp \beta)\right]\right\}$$

sustainable failure: $\frac{\partial \sigma_i}{\partial x} > 0$

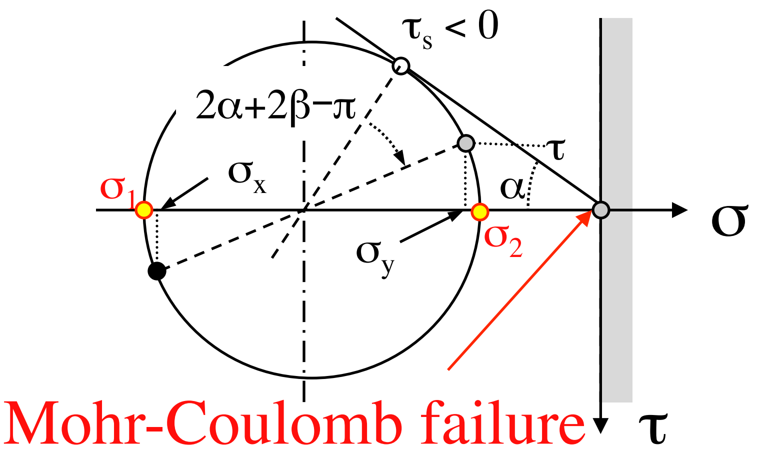
# Sign of $f_{\pm}$ sets avalanche sustainability

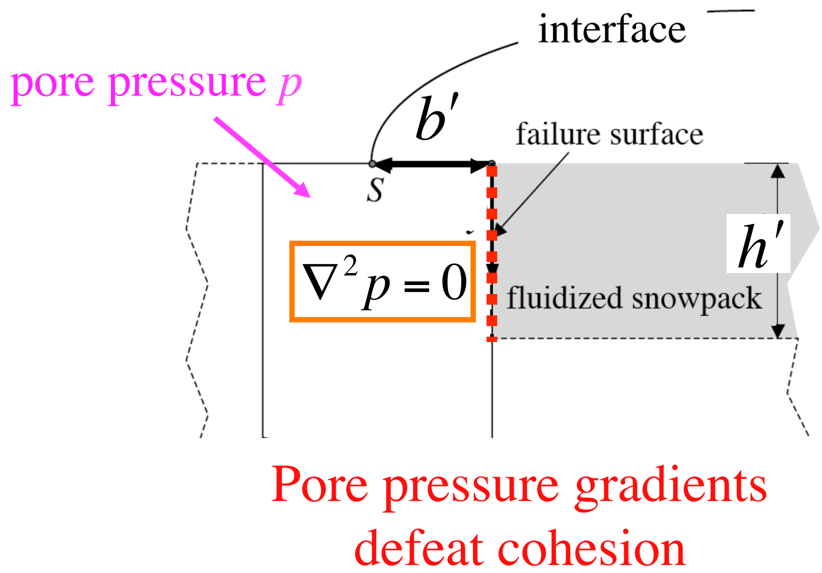



$$\tan\beta = \frac{F\cdot\hat{y} + \rho_c g}{F\cdot\hat{x}} \geq -\frac{1}{\mu_e}$$

Condition for sustainable failure, whence fluidized snow pack does not de-fluidize.

# Fluidization



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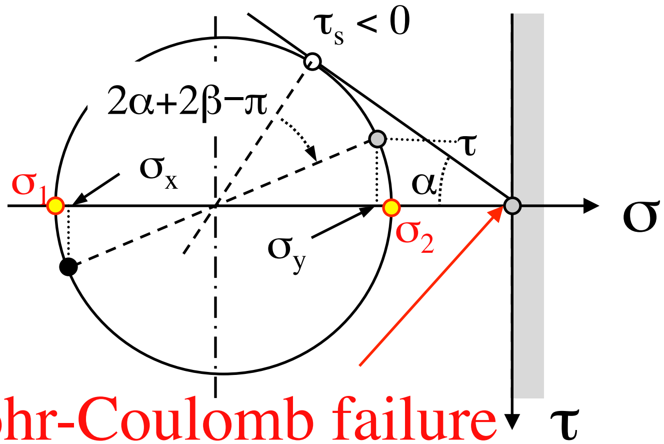
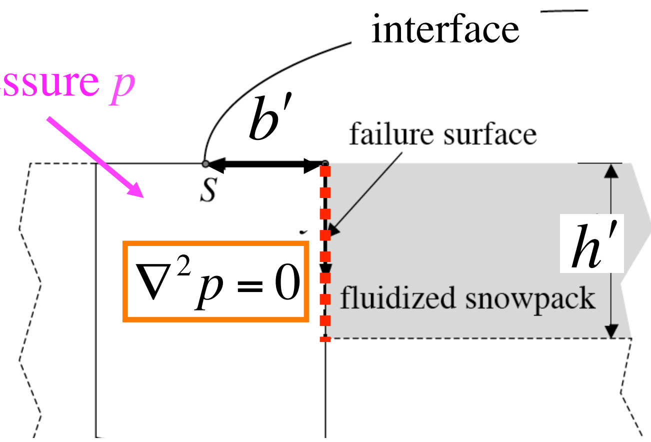



Mohr-Coulomb failure

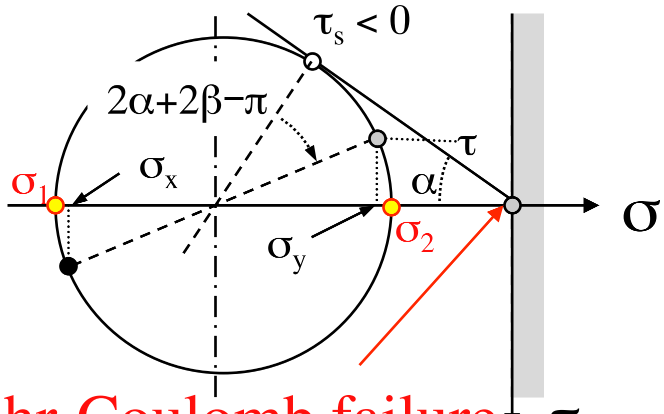



Pore pressure gradients defeat cohesion

$$R \equiv \frac{2\rho_c g b' \mu_e}{\rho' U'^2}$$

snowpack density $\rho_c$, friction $\mu_e$

# Fluidization

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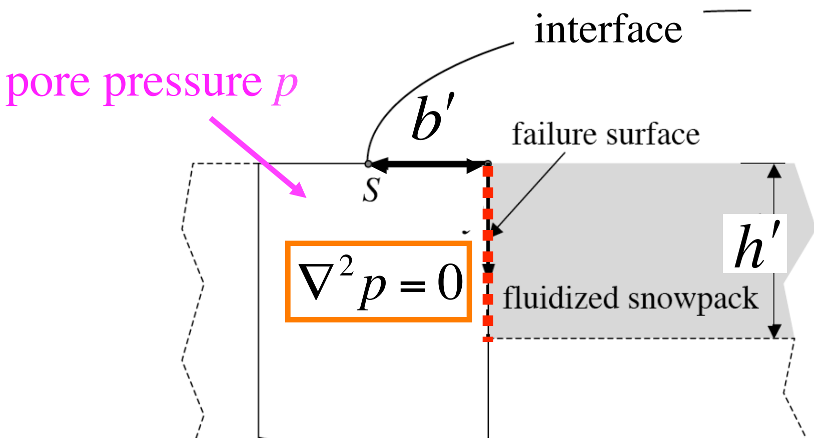


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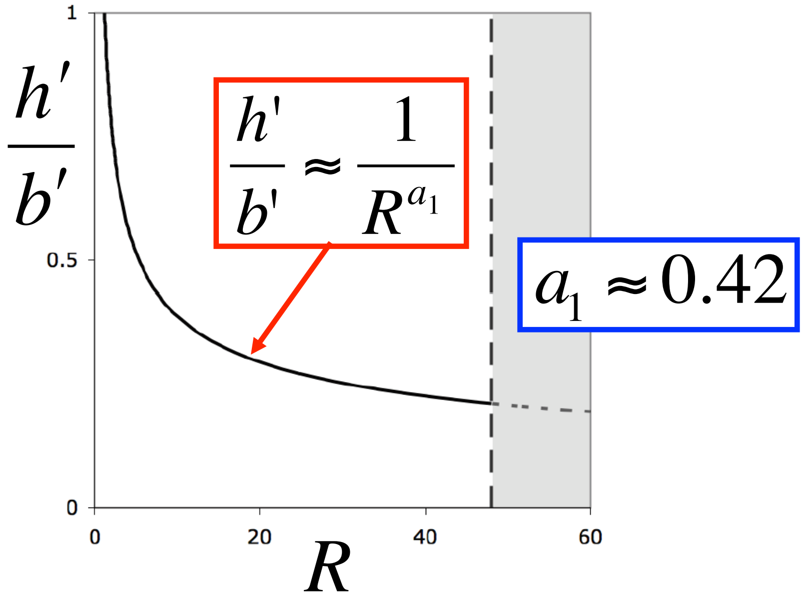


# *References*

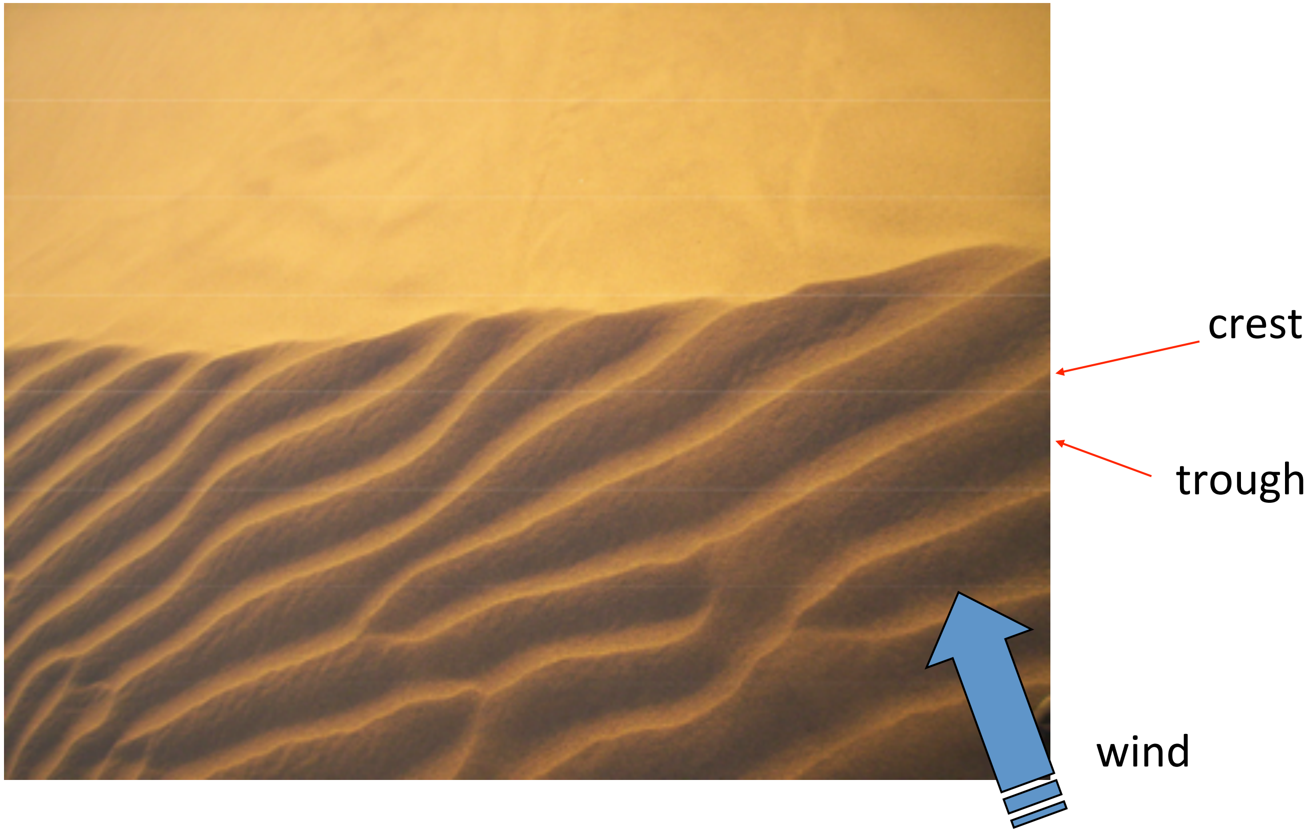
http://grainflowresearch.mae.cornell.edu

Louge, M. Y., C. S. Carroll, and B. Turnbull (2011), Role of pore pressure gradients in sustaining frontal particle entrainment in eruption currents: The case of powder snow avalanches, J. Geophys. Res., 116, F04030.

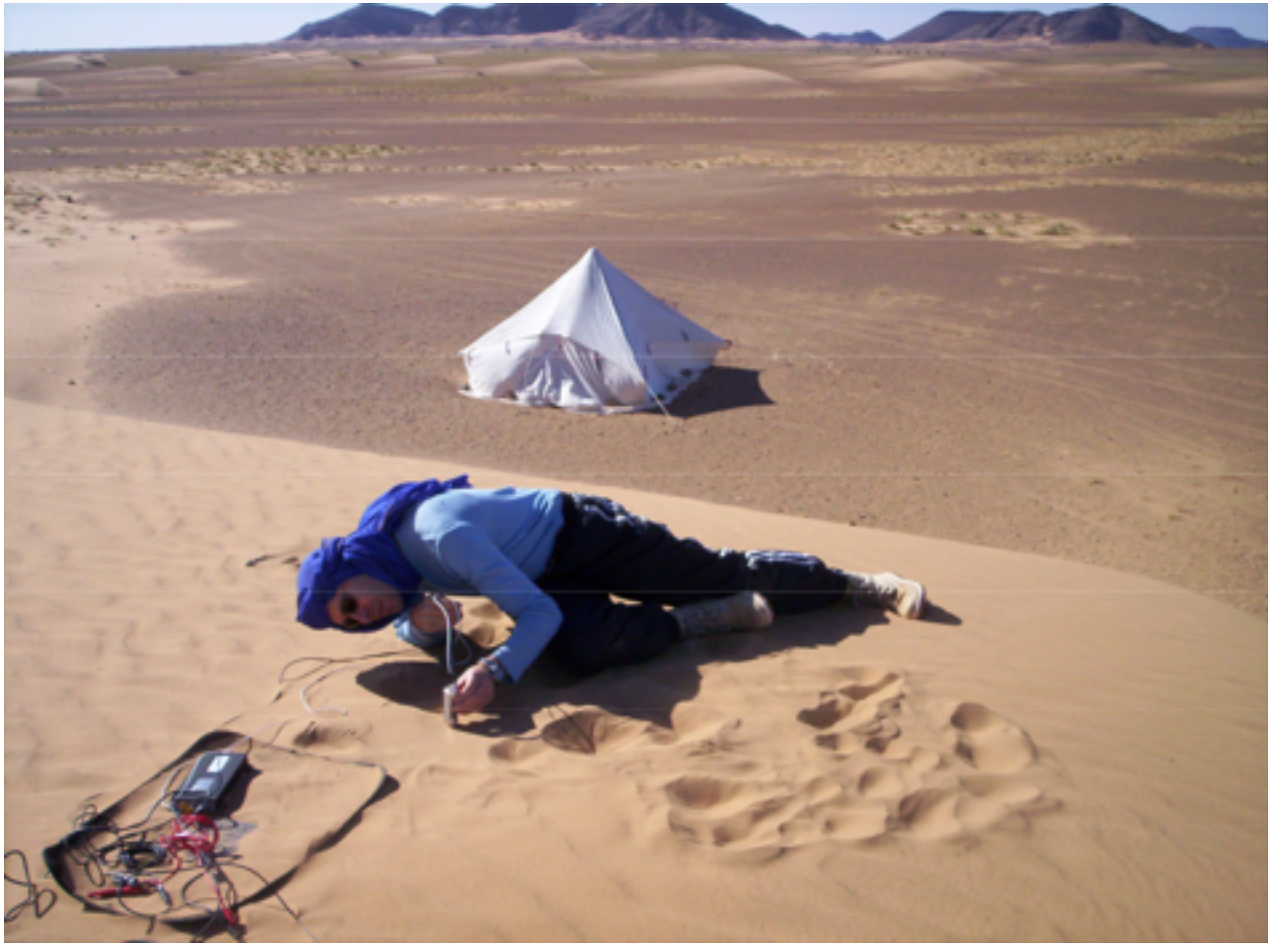
C. S. Carroll, B. Turnbull, and M. Y. Louge, Role of fluid density in shaping eruption currents driven by frontal particle blow-out, Phys. Fluids 24, 066603 (2012).

Carroll, C. S., M. Y. Louge, and B. Turnbull (2013), Frontal dynamics of powder snow avalanches, J. Geophys. Res., in press.

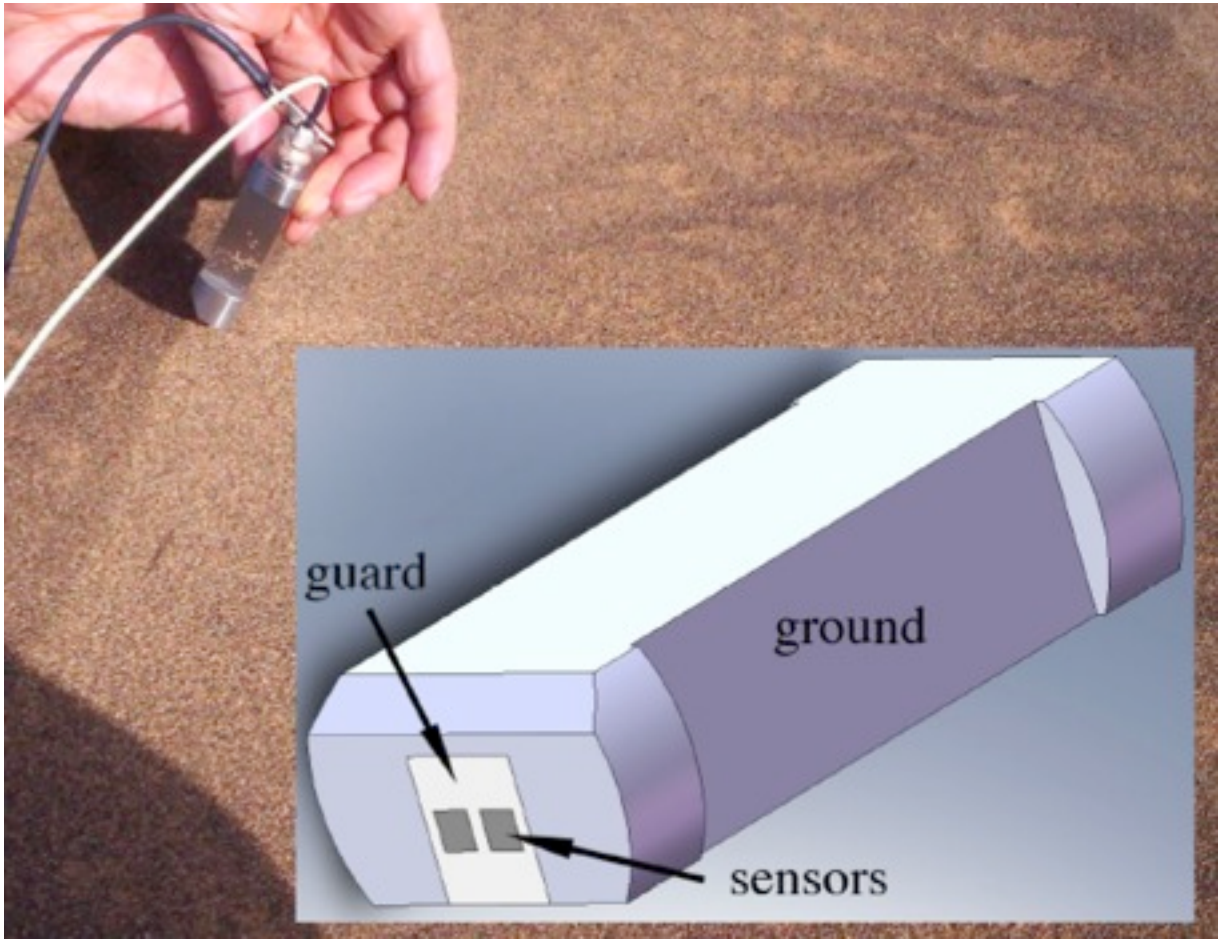
# Sand ripples



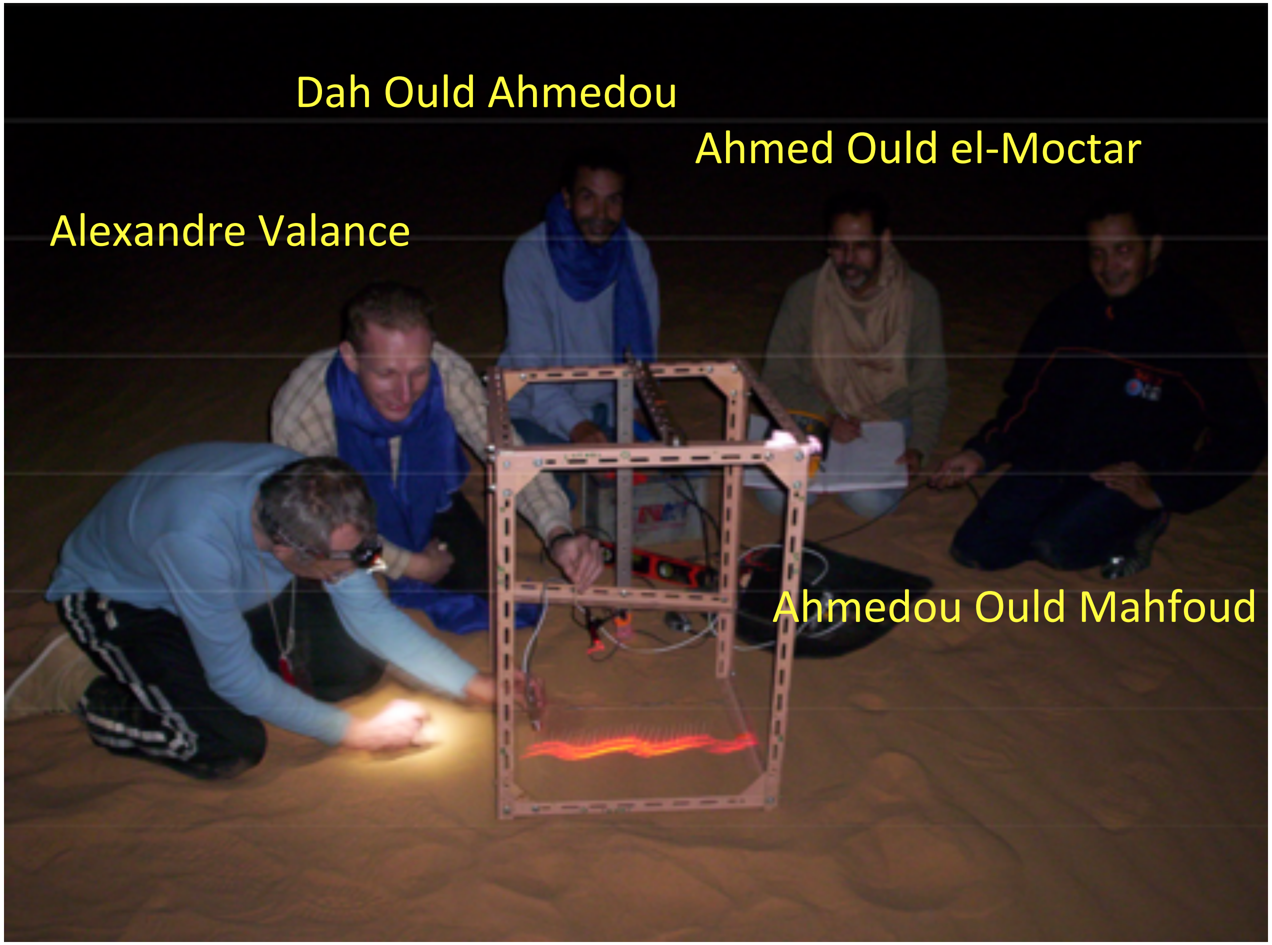
# Measurements



# Surface density probe

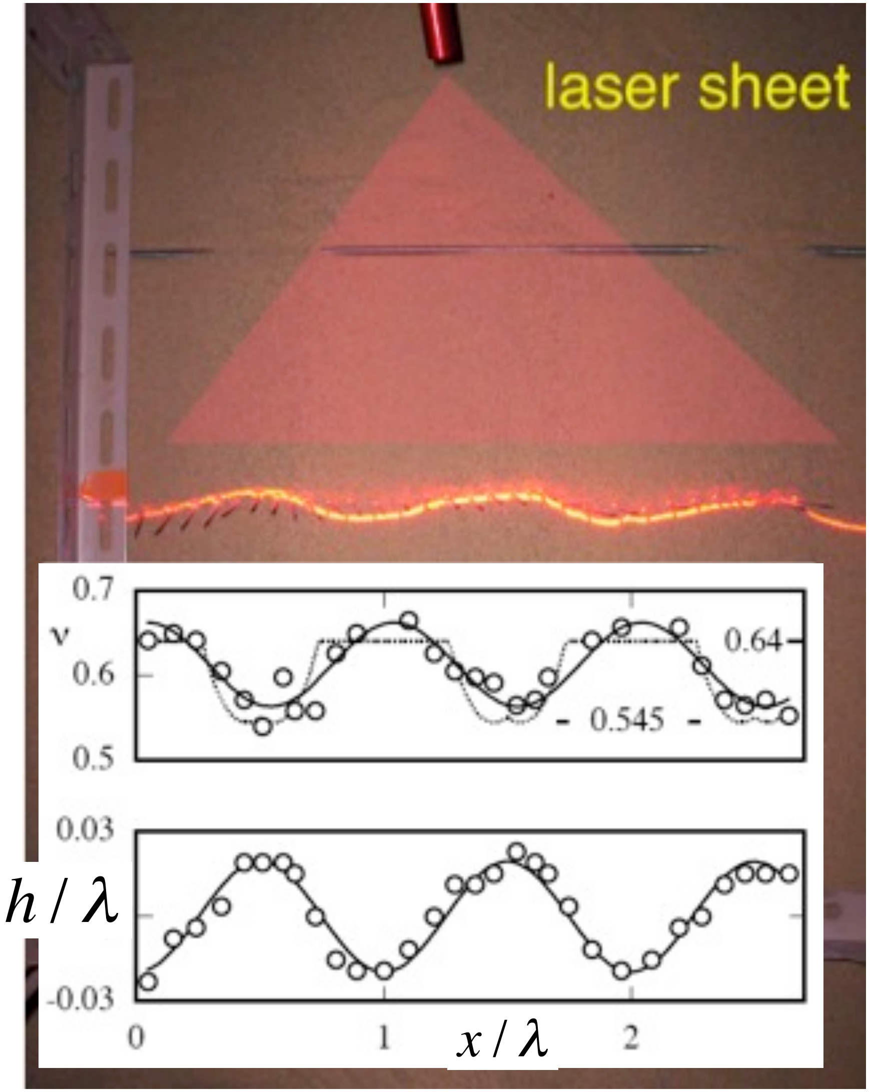


# *Ripple elevation profile*

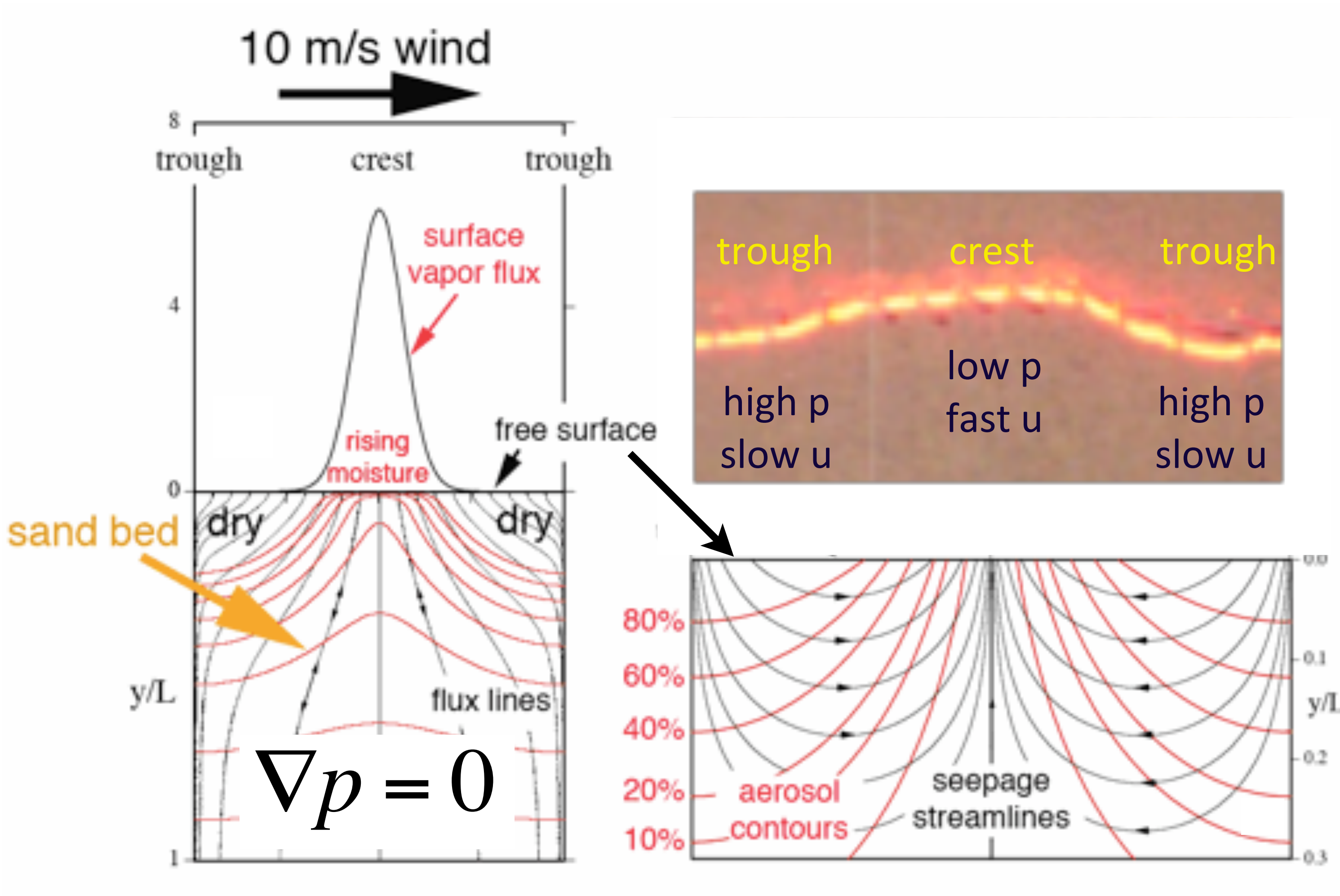



# *Elevation and volume fraction*

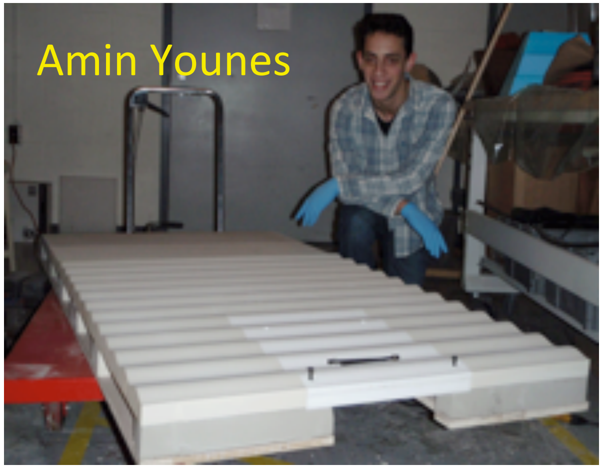
$$h_0 / \lambda \approx 0.02$$



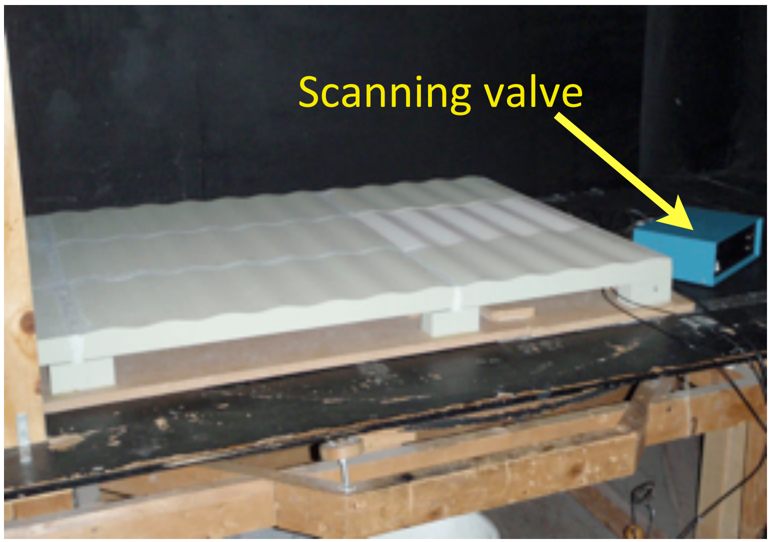
# Seepage, moisture and dust

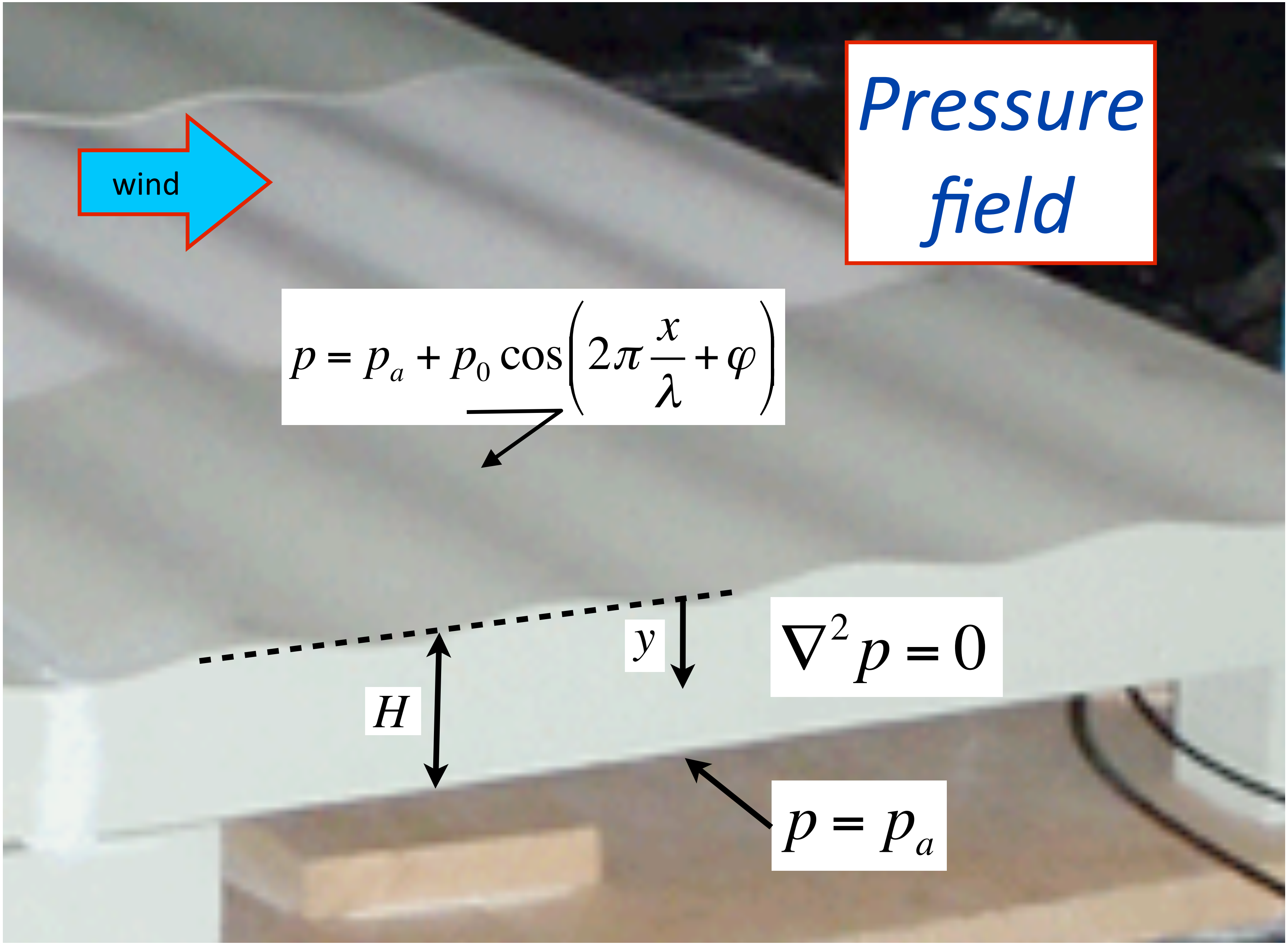


# *Wind tunnel experiments*

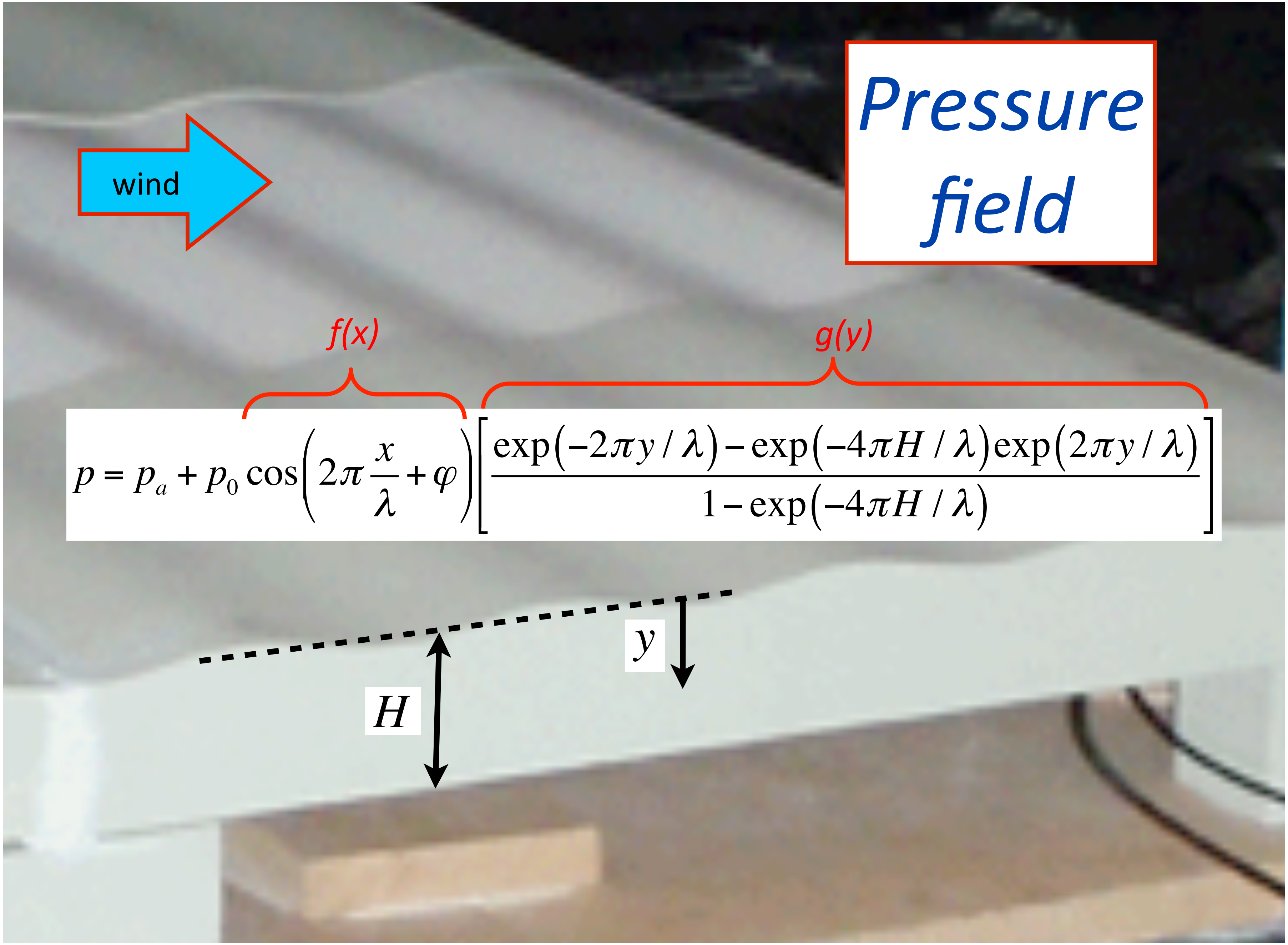






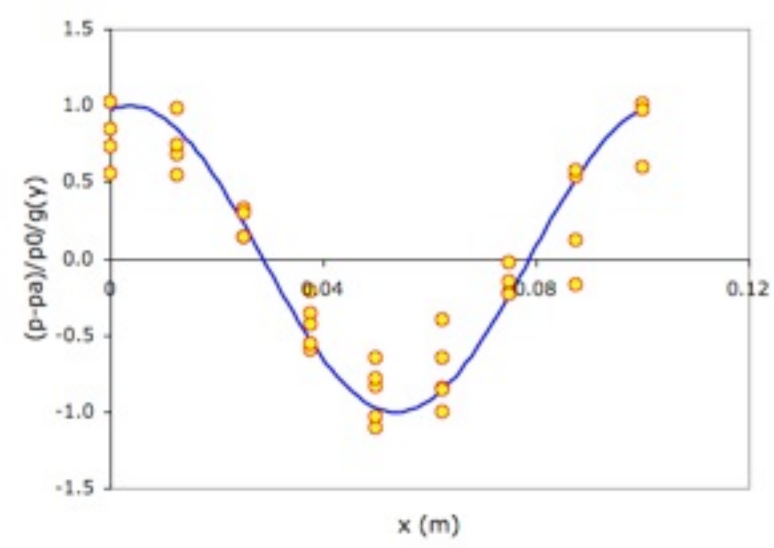


Pressure field
wind
$p = p_a + p_0 \cos\left(2\pi \frac{x}{\lambda} + \varphi\right)$
$y$
$H$
$\nabla^2 p = 0$
$p = p_a$

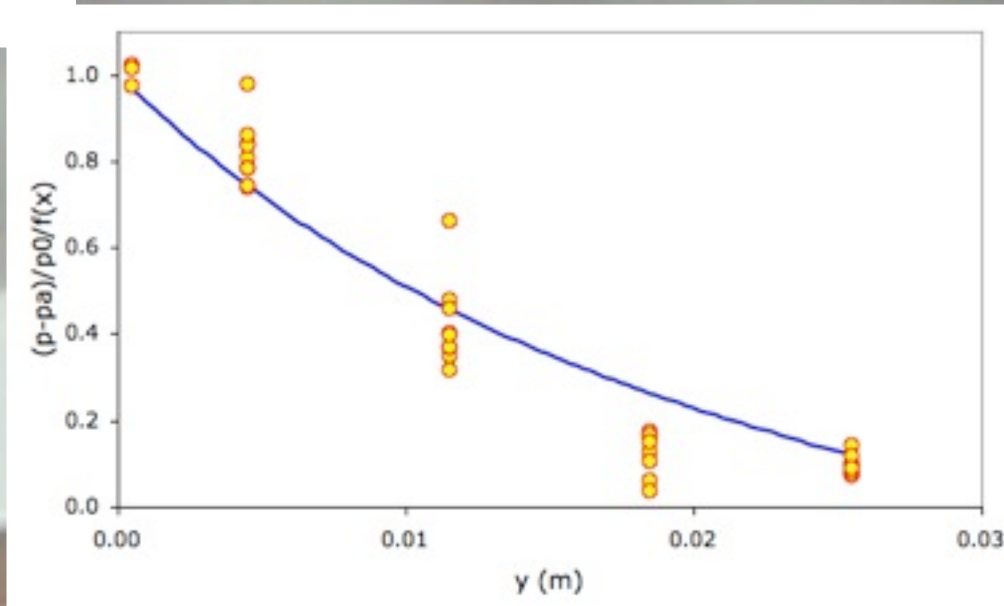
# Pressure field



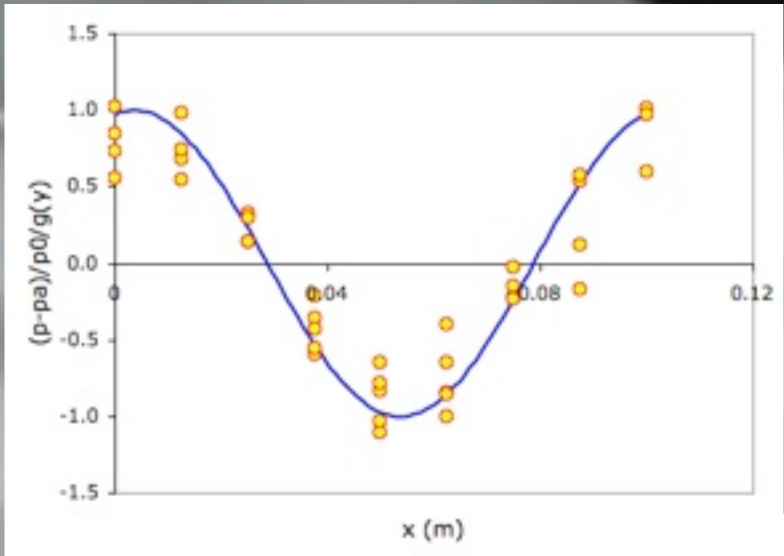

$$p = p_a + p_0 \cos\left(2\pi \frac{x}{\lambda} + \varphi\right)\left[\frac{\exp(-2\pi y/\lambda) - \exp(-4\pi H/\lambda)\exp(2\pi y/\lambda)}{1-\exp(-4\pi H/\lambda)}\right]$$

# *Pressure field*

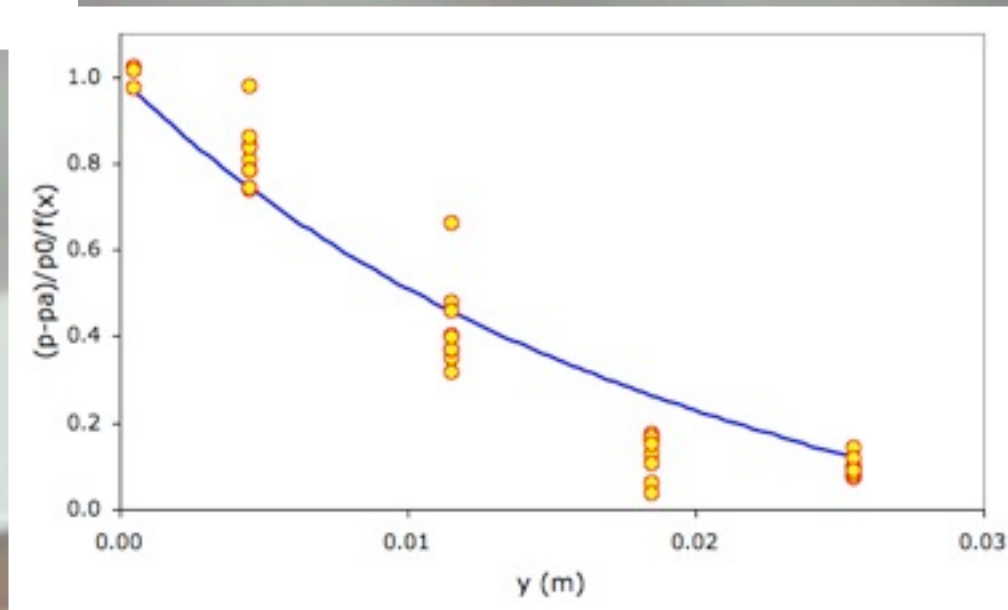


# *Pressure field*



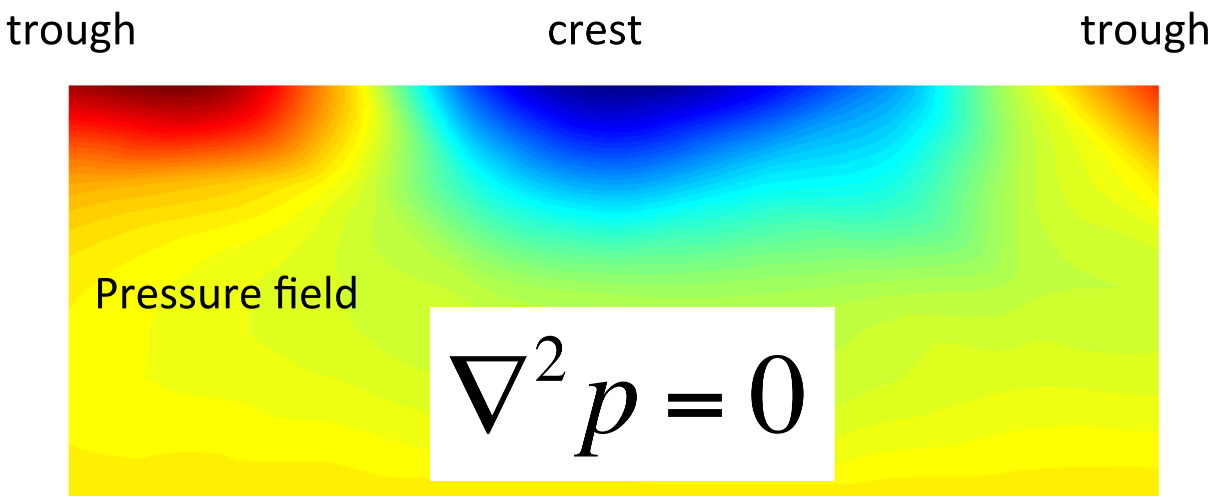

$$h_0 / \lambda = 3\%$$



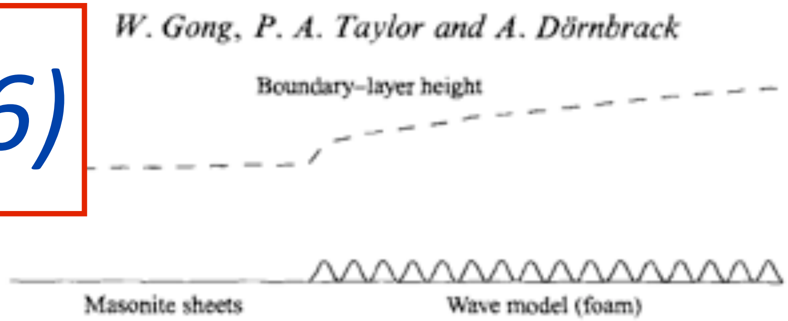
# *High amplitude to wavelength ratio*

$$p = p_a + p_1 \cos\left(\frac{2\pi x}{\lambda} + \phi_1\right) \frac{\exp\left(-\frac{2\pi y}{\lambda}\right) - \exp\left(-\frac{4\pi H}{\lambda}\right) * \exp\left(\frac{2\pi y}{\lambda}\right)}{1 - \exp\left(-\frac{4\pi H}{\lambda}\right)}$$
$$+ p_2 \cos\left(\frac{4\pi x}{\lambda} + \phi_2\right) \frac{\exp\left(-\frac{4\pi y}{\lambda}\right) - \exp\left(-\frac{8\pi H}{\lambda}\right) * \exp\left(\frac{4\pi y}{\lambda}\right)}{1 - \exp\left(-\frac{8\pi H}{\lambda}\right)}$$
$$+ p_3 \cos\left(\frac{6\pi x}{\lambda} + \phi_3\right) \frac{\exp\left(-\frac{6\pi y}{\lambda}\right) - \exp\left(-\frac{12\pi H}{\lambda}\right) * \exp\left(\frac{6\pi y}{\lambda}\right)}{1 - \exp\left(-\frac{12\pi H}{\lambda}\right)}$$

$$h_0 / \lambda = 6\%$$



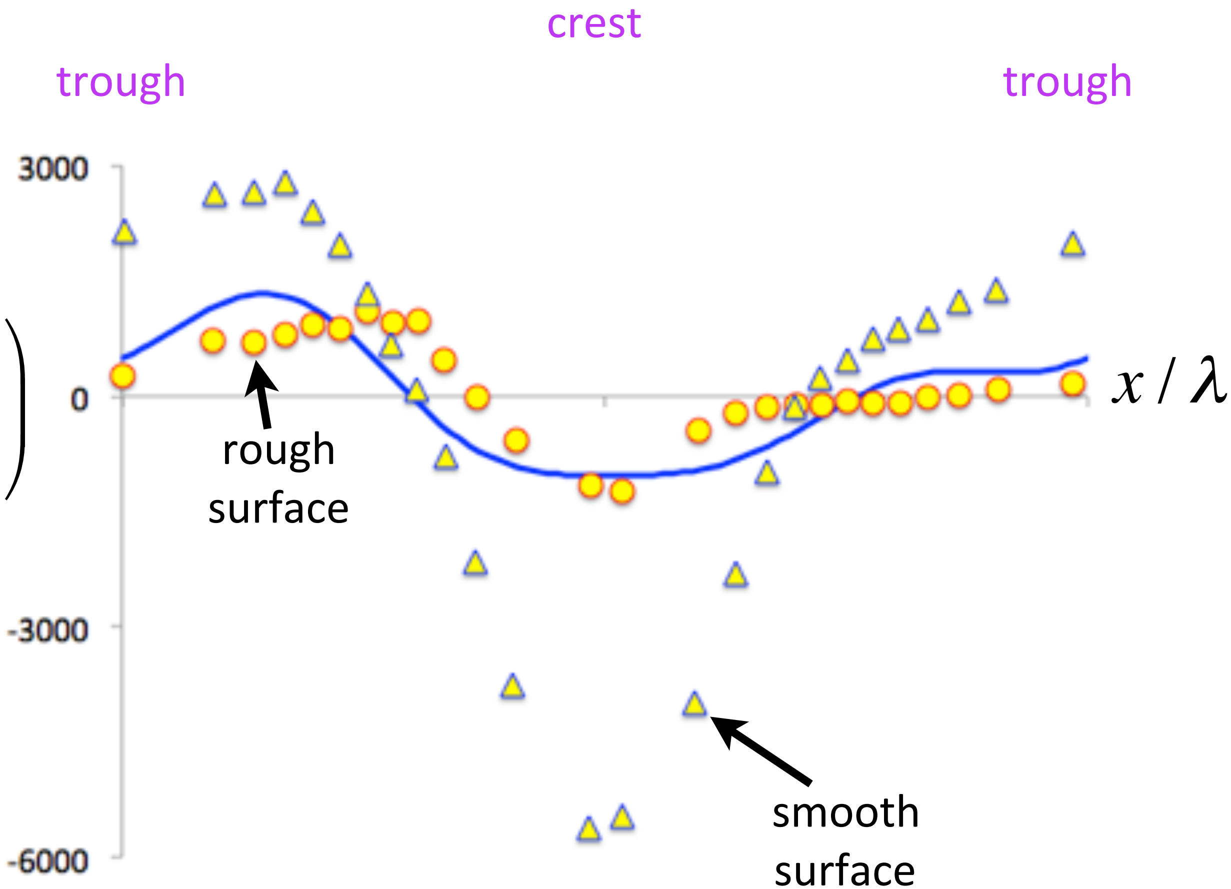
# Gong et al, JFM (1996)




$$u^* = 0.41 \text{ m/s}$$

$$h_0 / \lambda \approx 7.9\%$$

$$\frac{(p - p_a)}{\rho u^{*2}}\left(\frac{\lambda}{h_0}\right)$$



Kuzan, J. D., T. J. Hanratty, and R. J. Adrian, Turbulent flows with incipient separation over solid waves, Experiments in Fluids 7, 88-98 (1989).

# *Flow regimes*

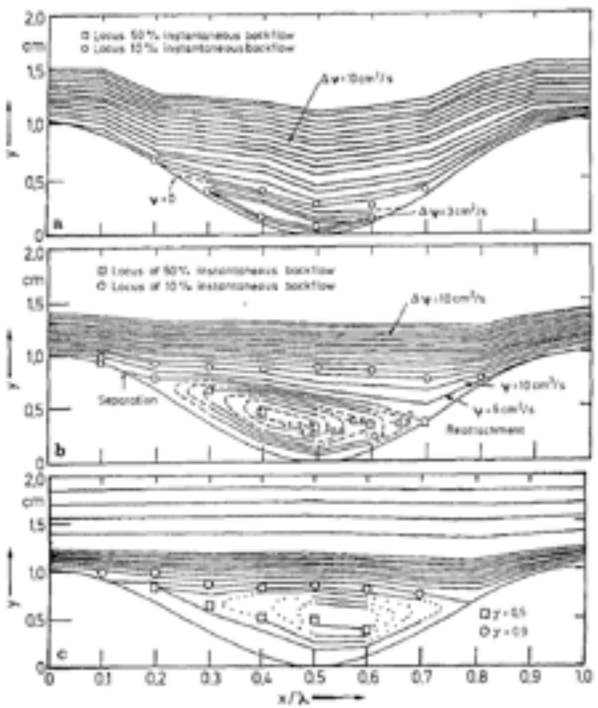
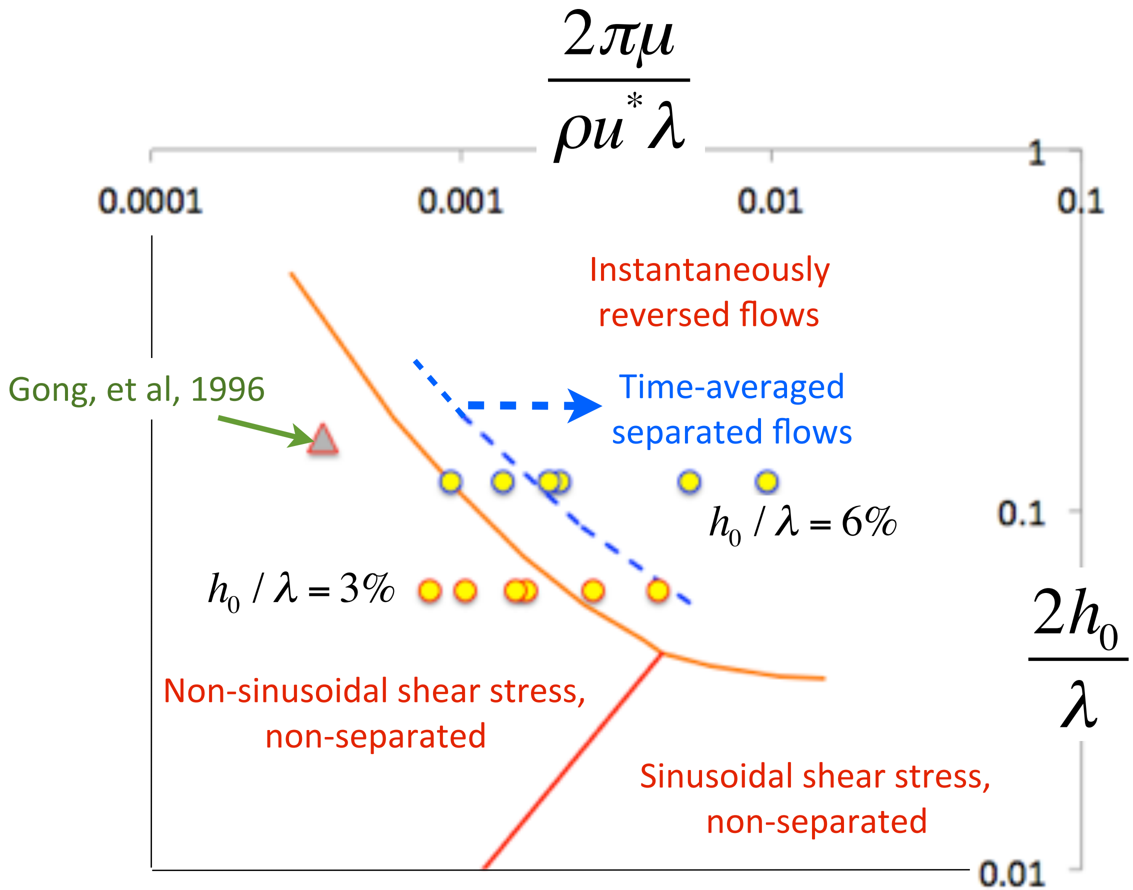
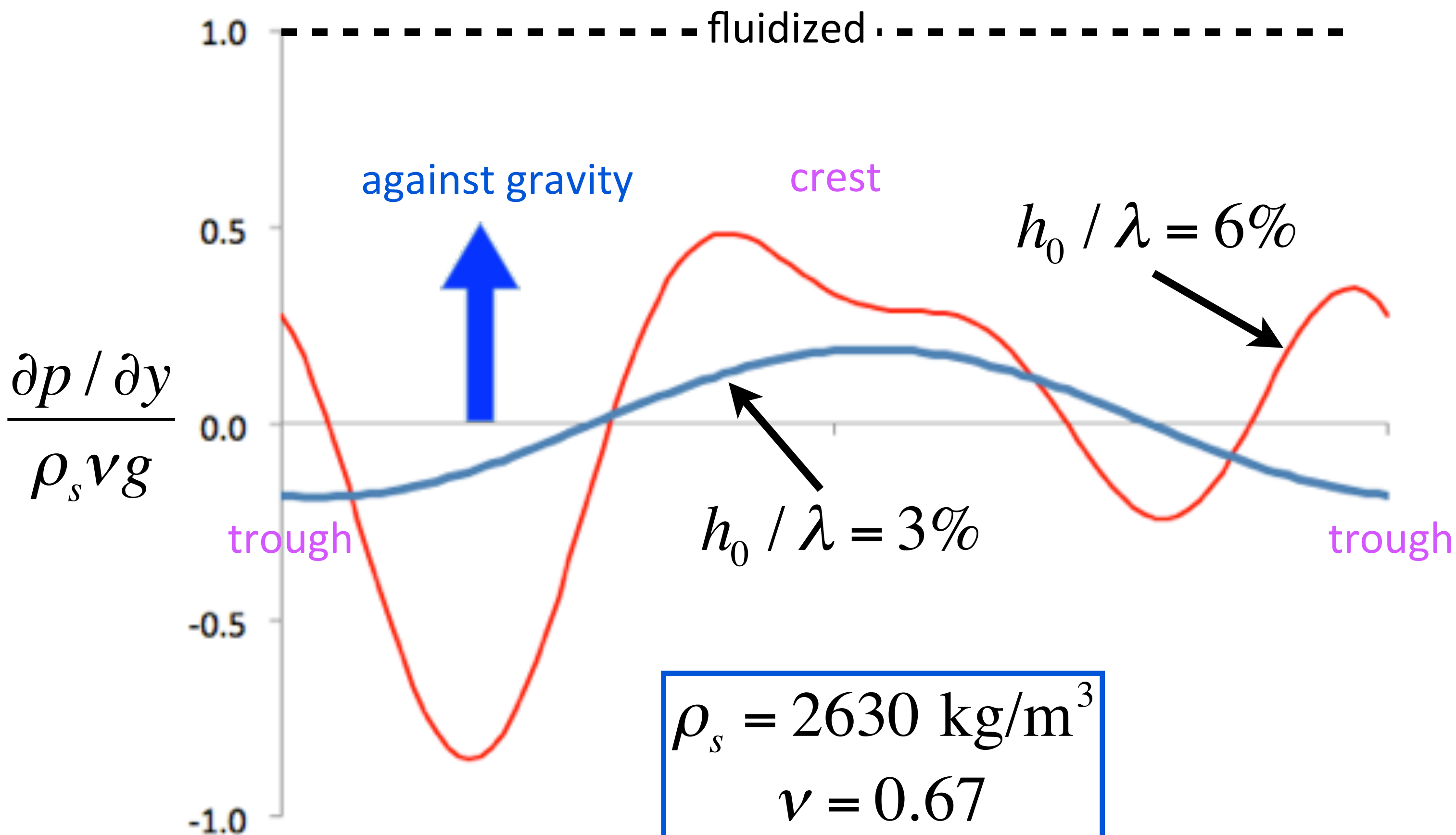



Fig. 19 a–c. Streamlines for $2a/\lambda = 0.2$, for a $Re = 4{,}000$, b $Re = 12{,}000$, and c $Re = 30{,}000$



# *Toward fluidization*




$\rho_s = 2630 \text{ kg/m}^3$

$\nu = 0.67$

$u^* \approx 1 \text{ m/s}$

# *References*


Louge, M. Y., A. Valance, A. Ould el-Moctar, and P. Dupont (2010),
Packing variations on a ripple of nearly monodisperse dry sand,
J. Geophys. Res., 115, F02001.

Louge, M. Y., A. Valance, H. Mint Babah, J.-C. Moreau-Trouvé,
A. Ould el-Moctar, P. Dupont, and D. Ould Ahmedou (2010),
Seepage-induced penetration of water vapor and dust beneath ripples and dunes,
J. Geophys. Res., 115, F02002.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