Packing variations on a ripple of nearly monodisperse dry sand

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Abstract. Dielectric measurements reveal that bulk density on the surface of recently-mobilized dry ripples of a nearly monodisperse desert sand varies from the random jammed packing at troughs to the minimum value for a stable packing at crests, suggesting that active ripples are relatively loose structures moving above a more consolidated substructure. Although the wind-driven evolution of static pressure on the surface induces air seepage through sand, we show that such internal flow is not strong enough to swell the bed and explain variations of bulk density on the surface.

1. Introduction

Wind mobilizes desert sands by overcoming static friction and other surface forces of van der Waals, electrostatic, or capillary origin. The resulting flow has low-energy grains “reptating” in a thin shear layer traveling on the sand bed, and fewer, faster-moving “saltating” particles accelerated by the free stream before impacting the reptating layer at a grazing angle [Bagnold, 1942; Andrott, 2004; Kok and Remno, 2008]. Such bombardment dislodges grains from the bed, brings them into reptation, and gives them initial agitation. Following Bagnold [1942], authors have also distinguished “creeping” grains rolling on the surface and “reptating” grains executing short hops after being ejected by impact with saltating sand in a process called “splash”. Recently, this distinction among low-energy grains has been blurred e.g., Manukyan and Prigozhin [2009]).

The formation, motion and rearrangement of ripples is a relatively slow process involving basal surface erosion, reptation, and gradual accumulation of sand, ultimately forming a pattern of troughs and crests perpendicular to wind blowing in the direction \(x\). Sand accumulation on ripple crests arises as streamwise variations of the solids mass flow rate \(\dot{m}'\) per unit width of the ripple crest line balance variations in the local surface erosion-deposition rate

\[
\dot{\phi}' = \frac{\partial \dot{m}'}{\partial x}.
\]

In turn, \(\dot{\phi}'\) sets the relatively slow rate of change of bed elevation \(h\) that creates ripples,

\[
\dot{h} = -\frac{\dot{\phi}'}{\rho_s \nu}.
\]

In this expression, \(\rho_s\) is the material density of sand, \(\nu\) is the solid volume fraction, the dot denotes the partial time derivative, and each prime indicates per unit length. Ignoring air mass, the solid volume fraction \(\nu = \rho_s / \rho_s\) or “packing fraction” is the ratio of bulk density \(\rho_b\) and sand material density \(\rho_s\). Its complement \((1 - \nu)\) is the “voidage”. Variations of \(\dot{m}'\) have been attributed to different rates of impact from saltating grains [Anderson, 1987] due to “shadowing” [Sharp, 1965], coupled with a BCRE model [Bouchaud et al., 1994; Csahok et al., 2000], to rolling and avalanching [Hoyle and Woods, 1997; Prigozhin, 1999], and to variations of the saltation flux along the ripple [Nishimori and Ouchi, 1993]. Predictions of the ripple pattern and its evolution are derived from non-linear stability analyses that model \(\dot{\phi}'\) [Csahok et al., 2000]. Manukyan and Prigozhin [2009] also proposed a numerical model capturing the role of a wide particle size distribution on ripple formation.

Ripple crests are often noticeably looser than the underlying sand surface. Meanwhile, the experiments of Rioual et al. [2003] suggested that the resuspension of bed particles is more likely to occur when saltating grains impact a loose assembly. Thus, streamwise variations of bulk density on the surface likely affect the bed erosion rate, a mechanism which Csahok et al. [2000] modeled in terms of local bed curvature. In this context, we bring insight into this process by quantifying variations of bulk density on the surface of a dry, recently mobilized sand ripple with a new capacitance instrument.

2. Instrument

We exploit the capacitance technique that Louge et al. [1997, 1998] developed to measure dielectric properties of the snow pack. In this technique, sketched in Fig. 1, a generic probe has three conductors, called the “sensor”, “guard” and “ground” electrodes. For our portable, self-contained electronics (Capacitec model 4100 with PC-201 preamplifier), an oscillator supplies a sinusoidal current of constant amplitude at a frequency \(f_s = 15\) kHz to the sensor using a feedback control circuit that maintains a stable voltage across a reference capacitance. A voltage follower of high input impedance samples the resulting sensor signal without disturbing it, driving the guard at precisely the same voltage. Crucially, the guard envelops the sensor everywhere except at the probe surface. To that end, the amplifier is surrounded by a guarded cage, and the probe is connected to the electronics using a high-quality coaxial cable with inner conductor and outer shield respectively at the sensor and guard voltages. The guard thus eliminates stray and cable capacitances, and permits the detection of capacitances as low as a few femtoFarads [Louge et al., 1996] (1 fF = \(10^{-15}\) F).

Calibrations using a parallel-plate capacitance held in front of a grounded target plate by a precision micrometer indicate that, if probes are exposed to a pure dielectric, the peak-to-peak guard voltage \(V_g\) of our own electronics is related to the capacitance \(C\) between sensors and ground using \(V_g \times C = Y \simeq 945\) V×fF. For robust desert operations, electronics are powered by a car battery and carried...
in a backpack. They are warmed up 30 min before operations, and their response is checked by holding a piece of glass with known dielectric constant in front of the probes.

We capture the oscillating guard voltage on a portable Fluke digital oscilloscope to find \( V_0 \) and the phase \( \phi \) of the guard with respect to the oscillator. Like snow, humid sand possesses a complex dielectric constant

\[
\epsilon_e = \epsilon'_e - \epsilon''_e,
\]

where \( i^2 = -1 \); the real and imaginary parts \( \epsilon'_e(\nu; \Omega) \) and \( \epsilon''_e(\nu; \Omega) \) are functions of \( \nu \) and the fraction \( \Omega \) of the total bed mass that is composed of water. \( \Omega \) is related to the “volumetric water content” \( \theta \approx \rho_w \nu/\rho_w \), where \( \rho_w \) is the density of liquid water. Thus, the complex impedance between sensors and ground is \( Z = [2\pi f_c C_0 (\epsilon'_e + \epsilon''_e)]^{-1} \), where \( C_0 \) is the capacitance of the probe held in air.

As Louge et al. [1997] showed, the Capacitec amplifier produces a voltage related to the modulus

\[
|\epsilon_e| = \sqrt{\epsilon'^2_e + \epsilon''^2_e} = Y/(V_0 C_0),
\]

and a phase that is related to the “loss tangent”

\[
\frac{\epsilon''_e}{\epsilon'_e} = |\tan \phi|.
\]

The probe of Figure 1 consists of two square sensors located symmetrically around the long axis of a rectangular guard. As Louge et al. [1998] calculated for a homogeneous medium, their measurement volume is bounded by two half-toroidal surfaces penetrating the bed a distance \( H = (a^2 - b^2)/2c \approx 11 \) mm perpendicular to the probe face, where \( 2a \approx 115.5 \) mm is the guard width in the direction joining the two sensors, \( 2b \approx 8 \) mm is the distance between exterior sides of the two sensors, and \( 2c \approx 1.6 \) mm is the width of the guarded gap separating them. The sensor thickness \( t_w \approx 3.1 \) mm on the long axis of the guard determines spatial resolution in the \( x \)-direction along the ripple. (Figure 2 of Louge et al. [1998] marks \( a, b, \) and \( c \) on a sketch of the probe head, but uses \( d \) to denote the thickness \( t_w \).) Nominally, the probe has a capacitance in air \( C_0 = (2ao t_w/\pi) \ln[(a + b)(a - c)/(a - b)/(a + c)] \approx 25 \) pF, where \( \epsilon_0 = 8.854 \times 10^{-12} \) F/m is the permittivity of free space.

![Figure 1. Probe recording profiles of bulk density on the surface of sand ripples. Because its guard has finite extent in the direction perpendicular to the line joining the sensors, its actual capacitance in air is \( C_0 \approx 40 \) pF.](image)

![Figure 2. Typical size distribution vs. diameter (\( \mu m \)) of collected particles, normalized with \( \int_{\mu m}^{\infty} PSD(\xi) d\xi \equiv 1 \).](image)

We evaluated properties of dry sands in the field by closely packing a sample to the volume fraction \( \nu \) in an open rectangular container of \( 50 \times 80 \times 40 \) mm\(^3\), by placing the surface probe on the dry sample, and by measuring \( \epsilon'_e \) at \( \Omega \approx 0 \). We calculated \( \rho_c \) and \( \nu \) by later weighing the sample and recording how much water volume it displaced. Following the suggestions of Louge et al. [1997] for non-spherical particles, we adopted the homogenization model of Böttcher [1952] to describe the dependence of the real part of the effective dielectric constant on \( \nu \),

\[
\frac{\epsilon'_e(\nu; \Omega) - 1}{3\epsilon'_e(\nu; \Omega)} = \nu \frac{\epsilon'_p(\Omega) - 1}{\epsilon'_p(\Omega) + 2\epsilon''_e(\nu; \Omega)}.
\]

which we used to extract the dry material dielectric constant \( \epsilon'_p(0) \) from the measured \( \epsilon'_e(\nu; 0) \) and \( \nu \) of the sample. We then used \( \epsilon'_p(0) \) to calculate volume fractions on dry beds from surface measurements of \( \epsilon'_e \). While measurements presented here concern dry sands at the dune surface (\( \Omega = 0 \)), our companion paper [Louge et al., 2009] will introduce another probe, and consider variations of humidity in the depth after establishing how the real part of the material dielectric constant \( \epsilon'_p \) varies with \( \Omega \).

3. Measurements

We recorded the evolution of bulk density along the surface of ripples on a small barchan dune holding nearly monodisperse sand (Figure 2) at the site studied by Ould Ahmedou et al. [2007] near Akjoujt, Mauritania, at 19°50.666′ N, 14°08.842′ W. Measurements were carried out on the convex “zone C” that Wiggs et al. [1996] identified between the brink and summit of such dunes. To insure that the surface was dry, and to allow optical profiling of ripple elevation, we operated shortly past sunset after the wind had stopped blowing. Figure 3 shows a photograph taken after completing measurements.

Profiling was achieved by intersecting the undulating surface with a thin triangular light sheet created by a cylindrical lens at the tip of a diode laser pointed \( \sim 30^\circ \) downward
along ripple crest lines. The trace was acquired by an overhead digital camera, and converted to ripple elevation after recording where a slender parallelepiped of known thickness intersected the sheet. The resulting ratio

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Our procedure was to place a light metal structure holding the laser and camera around the region of interest, and to take a photograph of the laser trace on undisturbed ripples. Then, we gently applied the capacitance probe of equation (A11) with $R = R_c$ at $y^+ = 0$. On the bottom graph, lines mark $h = \pm 3$ mm and intervals of 100 mm along $x$. The photograph shows the structure holding camera and diode laser, the light sheet intersecting the bed, and toothpicks planted where $h$ was recorded; the red triangle mimics spreading of the sheet.

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Figure 3. Surface profiles of $\nu$ (top graph) and dimensionless elevation $h^+ \equiv h/L$ (bottom graph) along the direction $x^+ \equiv x/L$ of prevailing winds blowing from left to right. Lines are best fits $\nu = 0.614 + 0.049 \cos(2\pi x^+ + \varphi_c)$ and $h^+ = -(h_0/L) \cos(2\pi x^+)$ with ripple amplitude $h_0 \approx 2.8$ mm and wavelength $L \approx 145$ mm; from the phase lag $\varphi_c \approx 195^\circ$, minimum $\nu$ occurs shortly behind the crest. The photograph shows the structure holding camera and diode laser, the light sheet intersecting the bed, and toothpicks planted where $\nu$ was recorded; the red triangle mimics spreading of the sheet.

spherical grains, and in good agreement with the measurements of Dickinson and Ward [1994]. At ripple crests, $\nu$ is approximately the random loose packing [Onoda and Liniger, 1990] or the so-called “melting transition” $\nu_m \approx 0.545$ of hard spheres [Zamponi, 2007].

The relatively large penetration $H \approx 11$ mm of the capacitance probe measurement volume in the vertical direction further suggests that loose packings are not confined to a thin region near the crest surface, but instead permeate the ripple. The qualitative observations in Figure 4 confirm this view. Here, one of us rammed a foot through a soft dune region exhibiting little or no cohesive material underneath. The ensuing stresses propagated through force chains in the bed [Radjai et al., 1998], and emerged exclusively from beneath ripple troughs, throwing particles vertically to a height of approximately 50 mm. This suggests that the material forming crests is made up of loose sand with $\nu$ below the “glass transition,” which occurs at a volume fraction $\nu_g \approx 0.575$ for hard spheres [van Megen et al., 1998]. Above $\nu_g$, sheared granular assemblies must dilate, as ripple troughs were forced to do in this simple test.

Figure 4. Successive photographs of the forceful impact of a foot against a soft, dry dune at 18°01.140′ N, 15°45.282′ W near Nouakchott, Mauritania, showing (a) the moment of impact; (b) as the foot sinks, sand is ejected from ripple troughs, (c) eventually reaching an altitude approximately 50 mm above the bed; and finally (d) the foot and sand at rest. The bottom close-up photograph shows aftermath of a similar event. The arrow shows where ejecta returned to ripple troughs whence they jumped. Note how crests appear unaffected by this process. The magnified region reveals a natural pattern of crosses marking the planes of Mohr-Coulomb failure. From their apex angle $2\alpha \approx 57 \pm 2^\circ$, we infer the internal friction $\tan \alpha \approx 0.543$ for this sand, see Appendix A. A short movie of this process is available on line.
4. Discussion

We expect surface variations of bulk density on dry ripples to matter in two ways. First, at troughs, higher volume fractions should raise the local threshold shear stress necessary to lift particles. Conversely, at crests, the looser grain assembly should facilitate the ejection of particles into reposition. This section discusses possible origins of these variations. We first consider the seepage of air through the porous bed that is driven by variations of static pressure induced by the ripple’s presence. Although Louge et al. [2009] predict that such seepage plays an important role in transferring dust and moisture through the surface, we dismiss its capacity to swell the bed and cause the variations of bulk density that we observed.

4.1. Role of seepage

As Buckles et al. [1984] and Gong et al. [1996] found in water and air, turbulent flows on wavy surfaces induce pressure oscillations due to streamline expansion and contraction. Günther and von Rohr [2003] reviewed experiments and theory for such flows. In a porous sand bed, these oscillations create a secondary gas “seepage” — air penetrates ripple troughs where pressure is highest, and re-emerges at crests, where it is lowest. A similar mechanism exists with underwater ripples [Meyssman et al., 2007; Huettel et al., 2007]. To establish whether seepage can cause variations of bulk density at the surface by swelling the bed, we first calculate the air pressure and velocity fields within the sand beneath a ripple.

We adopt a cartesian coordinate system with unit vectors $\hat{x}$ along the wind direction and $\hat{y}$ downward in the direction of gravity $g$. We consider ripples as two-dimensional objects with sinusoidal elevation $h = -h_0 \cos(2\pi x/L)$, amplitude $h_0$ and wavelength $L$. Because $h_0/L \ll 1$, we treat the bed surface as flat to simplify calculations within. Similarly, because $h_0$ is small compared to the seepage penetration $\sim L/2\pi$, we ignore surface variations of solid volume fraction reported in section 3 to calculate seepage pressure and velocity fields. Thus, we assume that the porous bed is made of identical cohesionless spheres of diameter $d$ at the uniform solid volume fraction $\nu = \nu_0 \approx 0.64$. We also take the seepage flow as steady. Its surface is subject to the static pressure profile

$$p(y = 0) \approx p_a + p_0 \cos(2\pi x),$$

where $p_a$ is the ambient pressure, and $p_0$ is its amplitude. Gong et al. [1996] observed

$$p_0 \approx \rho U^2 h_0/\eta L,$$

where $\rho$ is gas density, $U$ is wind speed, and $\eta \approx 0.14$. Gong et al. [1996] measured $U \approx 10 \text{m/s}$ at $Y \approx 1 \text{m}$ above the wavy surface, or at a wall coordinate $Y^+ = \rho Yu^*/\mu \approx 26,000$. (Such evolution of static pressure would also arise in a one-dimensional, steady, incompressible, inviscid and irrotational flow on a wavy surface $h = -h_0 \cos(2\pi x)$ at a distance $\eta L$ below a parallel flat wall).

In Appendix A, we verify that the seepage flow of air through the sand bed has a small Reynolds number. Therefore, it is subject to the viscous macroscopic Ergun’s equation

$$\mathbf{u} = -\frac{K}{\mu} \mathbf{V} p,$$

where $\mathbf{u} \equiv (1 - \nu) \mathbf{v}$ is the superficial gas velocity vector, $\mathbf{v}$ is the interstitial gas velocity,

$$K \approx \frac{\alpha^2}{150} \frac{(1 - \nu)^3}{\nu^2}$$

is the bed permeability, and $\mu$ is the gas viscosity. A consequence of equation (9) is that $\mu(1 - \nu)|(\nabla \nu) + (\nabla^* \nu)^2| \equiv 0$, so that gas shear stress vanishes identically within the porous bed. In principle, consistent with the Beavers and Joseph [1967] boundary condition, a thin boundary layer might form within the sand bed just beneath the surface. Because its thickness is on the order of $\sqrt{K} \approx 0.03d$, we ignore its existence. Thus, turbulence above the dune transmits no shear stress to the bed’s interstitial air, but rather to the sand bed assembly, and seepage is entirely driven by the static pressure profile at the bed surface.

For steady flow, the mass conservation equation is

$$\nabla \cdot \mathbf{u} = 0.$$

Therefore, in a homogeneous bed with negligible gas compressibility, pressure satisfies the Laplace equation $\nabla^2 p = 0$ subject to equation (7) at the surface and $p \rightarrow p_a$ as $y \rightarrow \infty$. Its solution is

$$p = p_a + p_0 \cos(2\pi x) \exp(-2\pi y),$$

yielding the local pressure gradient

$$\nabla p = -2\pi \frac{p_0}{L} \exp(-2\pi y) \{\sin(2\pi x) \hat{x} + \cos(2\pi x) \hat{y}\}.$$

and dimensionless velocity field

$$u' \equiv \mathbf{u}/(2\pi p_0 K) = \exp(-2\pi y)\{\sin(2\pi x) \hat{x} + \cos(2\pi x) \hat{y}\}.$$

Inspired by Earnes and Gilbertson [2000] and Mourques and Cobbold [2003], we now ask whether seepage can swell crests by exerting an upward drag defeating gravity and frictional forces. Appendix A reveals that, if the wind is strong, the sub-surface sand swells with a pattern of bulk density that is consistent with our observations on the surface (Figure 3, dotted line). However, it also shows that bed swelling is set by

$$R \equiv 2\pi p_0/(\rho, \nu, g, L),$$

which represents the fraction of gravity that seepage drag relieves. (Note that, although swelling does not explicitly depend on $d$, it is affected by grain size through $L$). For Akjoujt ripples, seepage only begins to defeat gravity ($R > 1$) for $U > 46 \text{m/s}$. Although it may release some grain contacts at typical wind speeds ($R \approx 5\%$ at $U = 10 \text{m/s}$), seepage does not swell the bed directly. Therefore, other explanations must be sought for streamwise variations of $\nu$ on ripples.

4.2. Ripple evolution

As this section suggests, ripple growth and evolution are likely responsible for packing variations observed on dry, recently mobilized, wavy surfaces of a nearly monodisperse sand. Using radioactive particle tracers in a wind tunnel, Barnardoff-Nielsen et al. [1985] noted that grains are periodically buried under a ripple, later re-emerging as the ripple moves forward [Barnardoff-Nielsen, 1986].

Grains advancing on the sand bed are moved by a surface shear stress $\tau_0$ that includes a contribution $\tau_0^\circ$ from air turbulence and another $\tau_0^\bullet$ from the momentum that saltation imparts on them by collisions. Because for ripples of small amplitude ($h_0 \ll L$) the total time-averaged shear stress is independent of altitude in steady, fully-developed flow, its magnitude $|\tau_0|$ at the surface is, to a good approximation, equal to its value above the particle cloud i.e., $|\tau_0| \approx \rho u^* \nu^2$, where $u^*$ is the turbulent shear velocity on a flat surface without particles, and the bar denotes average over time.
In general, both $\tau_0$ and $\tau_\infty$ can vary along the ripple. On a wavy surface, Gong et al. [1996] observed that the time-average air shear stress $[\tau_0]$ is high at crests and low at troughs, while boundary layer separation can occur on the downward ripple surface behind crests. Motivated to find a simple theory for the dynamics of barchan dunes, Kroj et al. [2002] modeled the evolution of $[\tau_0]$ along a wavy surface by simplifying the treatment of Hunt et al. [1988]. They expressed the stress in terms of local surface slope and an integral convolving slopes at distances effectively $\lesssim o(L)$ away, $[\tau_0](x) \sim \rho u^2[1 + B \partial h/\partial x + A \int_{-\infty}^{\infty} \partial^2 h/\partial^2 x] \partial h(x - \xi)/\partial x$, where $A$ and $B$ are adjustable constants [Hersen, 2004]. Manukyan and Prigozhin [2009] adopted a similar framework to calculate the evolving shear stress on a rippled surface.

While $\tau_0$ is chiefly determined by nearby surface curvature, $\tau_\infty$ is affected by saltation, which borrows momentum from the wind farther away, and thus should be more uniformly distributed along $x$ at the ripple scale [Manukyan and Prigozhin, 2009]. However, where the incident inclination of saltating particles is less than the downward slant of the ripple surface, such as right behind steep crests, Sharp [1965] noted that impact “shadowing” protects surface grains, thus effectively canceling the local contribution of saltation to $\tau_\infty$. More generally, the way by which saltation imparts stress on surface grains should depend on local bed inclination and packing. This concept is consistent with the standard explanation for growth of aeolian ripples on an initially flat, erodible bed, whereby saltating particles impact the ripple’s windward slope at an angle closer to the outward normal than the downwind slope, thus ejecting more grains into reptation, and producing a greater mass flow rate $m$ that pushes grains toward the ripple crest [Bagnold, 1953; Anderson, 1987]. In short, $\tau_0$ and $\tau_\infty$ both vary along $x$, thus affecting the local rate at which reptating grains are mobilized.

In this context, for thin granular flows driven by a surface shear stress, the depth-averaged solid volume fraction decreases as surface shear stress increases. Conversely, when this stress falls anywhere below an entrainment threshold, $\tau_\infty$ can vary along the ripple surface by saltation imposition of a turbulent gas). Meanwhile, lower values of $\tau_\infty$ can arise behind ripples crests if $\tau_0$ collapses by boundary layer separation and/or $\tau_\infty$ does so by salting shadowing. Such mechanisms might explain how the radioactive grain tracers of Barnsford-Nielsen et al. [1985] could have been trapped by advancing ripples leeward of their crests.

In general, because free-surface granular flows are prone to jamming at solid volume fractions $\gtrsim 0.6$, they dilate while being sheared, often maintaining a volume fraction 0.545 $< \nu < 0.595$ [Silbert et al., 2000]. We expect that mobile grains preserve such relatively loose fabric once trapped in a ripple. Because ripples are ephemeral objects without significant overburden, few mechanisms can pack them as they move, or for some time after they stop. As ripples travel slowly forward, compaction may arise by impact bombardment from saltating grains. An order of magnitude analysis comparing this process to packing by horizontal vibration suggests that periodic bombardment and shear stress fluctuations cannot result in any significant compaction deep within the ripple: in a unit length along the crest, the mean horizontal fluctuating force is $\sim o(\tau_0 L)$, acting on a ripple mass $\sim \rho_v v_0 L^2$; thus the relative acceleration is $\Gamma \sim \rho a^2(\rho_v \nu h u g)$ [Ristow et al., 2003]; for Akjoujt ripples, $\Gamma \sim 0.004$, which is much smaller than the minimum value $\Gamma \sim 0.01$ at which shaking begins to compress a granular assembly [D’Anna and Gremaud, 2001]. Nonetheless, while saltation bombardment is unlikely to pack ripples in depth, it is possible that surface grains might rearrange more closely upon repeated impacts.

A longer-term compaction mechanism is due to thermal cycling through sand of thermal diffusivity $\alpha$ over several days of $J = 24$ hrs. Although grains in a mobile, fully-developed ripple are everywhere within the depth $\sim \sqrt{\alpha J}/\pi \approx 0.07$ m through which diurnal temperature variations matter [Carslaw and Jaeger, 1959], a typical ripple turn-over time $\sim 10^5\sqrt{d/g} \approx 450$ s [Andreotti et al., 2006] is $< J$, too short to pack recently mobile ripples. However, because the basal surface exposed by the ripple trough endures much longer, particularly near the dune apex where sand erosion and deposition rates balance, this surface may be subject to enough thermal cycles to explain its high packing $\sim \nu_*$, as the recent experiments of Devoux et al. [2008] suggest. Consistent with this, we report in the companion paper [Louge et al., 2009] that packings remain near $\nu_* \approx 0.64$ within the first few cms below the dune surface, in sharp contrast with the loose packing recorded beneath crests (except at the avalanche face and at point 3 just above the horn, where packing is loose). The observations of Fig. 4 further support this view: ramming forces are chiefly transmitted to ripple troughs, suggesting that the latter form a relatively compact base connected by the “strong force network” of Radjai et al. [1998]; conversely, ripple grains loosely deposited beneath crests likely constitute a “weak force network.”

Because our observations were carried out in the field on a nearly monodisperse, recently mobilized sand, they might not represent ripples with wider size distribution. For example, “giant” ripples, which are armored by larger grains rolling on the surface and accumulating at crests [Tsour, 1990; Manukyan and Prigozhin, 2009], are less mobile, perhaps giving the ripple enough time to compact, and their shape is no longer harmonic.

However, our observations and those of Barnsford-Nielsen et al. [1985] suggest that, in an active ripple of narrow size distribution, the region with elevation between trough and crest is filled with loose, recently mobilized sand. From this view, it is tempting to speculate that a ripple of nearly monodisperse, round, dry sand is a loose, slowly advancing object that “rolls” on a relatively passive denser substratum under the action of turbulent air and saltation bombardment, repeatedly burying and dislodging the same population of reptating grains as it progresses.

5. Conclusions

Dielectric measurements on a barchan dune revealed that bulk density at the surface of a dry ripple of nearly monodisperse round sand varies from random jammed packing at troughs to random loose packing at crests. The kicking of a soft sand bed also suggested that dry crests are relatively thick compliant regions of low density unable to transmit significant stress.

We introduced a model of the internal seepage flow driven by variations of static pressure on the surface of a wind-swept ripple. An analysis of the frictional sand bed showed that the resulting seepage drag contributes to relieving gravity, but is not able to swell the bed beneath crests to the observed random loose packing at reasonable wind speed. Instead, the ripple picture that emerges is one of a hard, almost immobile flat base (the trough), perhaps compacted by diurnal thermal cycles, and a heap (with crest) slowly advancing at the minimum bulk density for a stable solid. Further measurements are needed to confirm speculation that ripples of monodisperse dry sand are objects executing a slow rolling motion on a hard base.
Appendix A: Seepage and bed expansion

We show that, for typical wind speed, the volumetric seepage drag working against gravity is insufficient to expand the interconnected granular assembly to a lower solid volume fraction. To that end, we ignore the possible role of repletion in helping fluidize the bed, and we adopt pressure variations from equation (12). Because seepage has negligible inertia, its momentum conservation reduces to a balance between the drag force $F_d$ exerted by the gas on solids in a unit volume and the local pressure gradient exerted on the fraction of the volume occupied by the gas, $-\Delta Vp = F_d$.

Then, the total force $F$ exerted by the gas on solids is the sum of $F_d$ and the buoyancy force arising from the gas pressure gradient, $F = F_d - \Delta Vp = -\Delta Vp = \mu u/K$ [Anderson and Jackson, 1967]. Because $F \propto K^{-1}$ and the superficial gas velocity $u$ is conserved, a small variation of $v$ away from $u_c$ corresponds to $F(v) \approx F(u_c)[(\nu/v)/K(v)]$, where $\nabla p(v_c) = -F(v_c)$ is calculated at $v = v_c$. Thus,

$$F = -\frac{K(u_c)}{K(v)} \nabla p,$$  \hspace{1cm} (A1)

where $\nabla p$ is given by equation (13). Assuming that the bed is a homogeneous continuum with symmetric plane stresses, balancing forces on solids along $\hat{x}$ and $\hat{y}$ yields, respectively,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} + F \cdot \hat{x} = 0$$  \hspace{1cm} (A2)

and

$$\frac{\partial \tau}{\partial x} + \frac{\partial \sigma_y}{\partial y} + F \cdot \hat{y} + \rho g v = 0,$$  \hspace{1cm} (A3)

where $\sigma$ denotes a normal stress on surfaces perpendicular to the index shown, and $\tau$ is the shear stress on these surfaces. In these equations, we adopt the convention that compressive normal stresses are negative, and that the shear stress on a surface of normal $\hat{y}$ directed along $\hat{x}$ is positive. Solving equations (A2) and (A3) requires constitutive stress-strain relations and boundary conditions on all sides.

However, bed expansion can be modeled without specifying granular constitutive laws and solving equations (A2) and (A3) formally in the periodic interval $x \in [0, L]$ and $y \in [0, \infty)$. To that end, without loss of generality, we focus on the region beneath the free surface ($y > 0$) with $L/4 < x < L/2$. (A mirrored region of similar behavior is located across the plane $x = L/2$). There, the $y$-body force $F \cdot \hat{y} + \rho g v$ may turn the pressure of tensile stresses ($\sigma > 0$), which are prohibited if the granular material is not cohesive.

Where equations (A2) and (A3) would predict such tensile stresses, we postulate that the granular assembly expands to avoid them, while exhibiting incipient Mohr-Coulomb failure on slip planes with the same orientation relative to the resultant body force than the surface of a reposing heap relative to gravity. This implies that, in the expanded bed, a slip plane has a normal directed an angle $\alpha$ counterclockwise from the resultant body force $F + \rho g v \hat{y}$ if the shear stress $\tau_s$ on such plane is negative (Figure 5a), and clockwise if it is positive (Figure 5b). We identify $\alpha$ with the angle of repose, and equate it to the internal friction coefficient $\mu_E \equiv \tan \alpha$.

The signs of $\tau_s$ imply two possible states of stress for the granular assembly in the expanded bed. Figure 5 illustrates them both. For $\tau_s > 0$, we find

$$\sigma_x = \sigma_y \left[ 1 - \sin \alpha \sin(3 \alpha + 2 \beta) \right] \frac{1 + \sin \alpha \sin(3 \alpha + 2 \beta)}{1 + \sin \alpha \sin(3 \alpha + 2 \beta)},$$  \hspace{1cm} (A4)

and

$$\tau = \pm \sigma_y \left[ 1 - \sin \alpha \sin(3 \alpha + 2 \beta) \right] \frac{1 + \sin \alpha \sin(3 \alpha + 2 \beta)}{1 + \sin \alpha \sin(3 \alpha + 2 \beta)},$$  \hspace{1cm} (A5)

where $\beta$ is the angle of the resultant body force with $\hat{x}$.

$$\tan \beta = \frac{F \cdot \hat{y} + \rho g v}{F \cdot \hat{x}}.$$  \hspace{1cm} (A6)

An analysis of equation (A5) reveals that

$$(\tan \alpha - 1/\sqrt{3}) \times \tau_s < 0 \times \tan \beta < 0, \forall \alpha.$$  \hspace{1cm} (A7)

For small winds, this condition implies $\tau_s < 0$ when $\tan \alpha < 1/\sqrt{3}$ and $\alpha > 0$ otherwise.

We substitute these expressions in equations (A2) and (A3), assume that $\beta$ varies slowly, combine them linearly to $\tau = F \cdot \hat{y} + \rho g v$

$$(\tan \alpha - 1/\sqrt{3}) \times \tau_s < 0 \times \tan \beta < 0, \forall \alpha.$$  \hspace{1cm} (A7)

For small winds, this condition implies $\tau_s < 0$ when $\tan \alpha < 1/\sqrt{3}$ and $\alpha > 0$ otherwise.

Figure 5. Mohr-Coulomb circle. Normal stresses $\sigma_x$ and $\sigma_y$ on planes perpendicular to $\hat{x}$ and $\hat{y}$ are on the abscissa; shear stress $\tau$ is on the ordinate along the positive direction shown. Open circles represent the plane on which the granular assembly is postulated to fail with internal friction $\mu_E = \tan \alpha$. The lightly and heavily filled circles are planes of normal $\hat{x}$ and $\hat{y}$, respectively.

Figure 6. Diagram of $\tan \beta$ (equation A6) vs. internal friction $\tan \alpha$. Dark shading indicates regions where $\zeta (\tan \beta, \tan \alpha) < 0$. The diagram is plotted by upholding condition (A7) at low winds; therefore, $\tau_s$ must be negative to the left of the vertical dash-dotted line, and positive to the right. As wind strengthens, $\beta$ moves downward in this diagram on a vertical line at fixed $\tan \alpha = \mu_E$ from $\beta = +\pi/2$ (no wind) until it reaches the boundary of the shaded region (solid line) where $\beta = \beta_0$. For stronger winds, $\beta$ remains equal to $\beta_0$ while the bed swells.
eliminate $\partial \sigma_y / \partial x$, define $\sigma^\dagger \equiv \sigma / (\rho \nu g L)$, and find

$$
\frac{\partial \sigma_y^\dagger}{\partial y} = -\frac{R}{K(\nu)} \left( (1 + \sin \alpha \sin(3\alpha \mp 2\beta)) \left( \frac{(1 + 4 \sin^2 \alpha)}{1 + \tan^2 \beta} \right) \frac{\zeta(\tan \beta, \tan \alpha)}{\tan \beta} \right),
$$

(A8)

where

$$
R \equiv 2 \rho u_0 / (\rho \nu g L)
$$

is the dimensionless group governing this system, having the structure of a Shields parameter. We define the function

$$
\zeta(\tan \beta, \tan \alpha) \equiv (\tan \beta \pm \tan \alpha) \left( \tan \beta - \tan \beta^\dagger \mp \tan \beta^\dagger \right),
$$

(A10)

with

$$
\tan \beta^\dagger_j \equiv \frac{-2 \tan \alpha(1 - \tan^2 \alpha) \mp (-1)^j \sqrt{2}}{1 + 5 \tan^2 \alpha}
$$

(A11)

for $j = 1$ or 2, and $\xi \equiv (2 \tan \alpha - 1)(1 + 2 \tan \alpha + 2 \tan^3 \alpha + \tan^2 \alpha + 2 \tan^5 \alpha)$. Because $\sigma^\dagger$ nearly vanishes at the free surface, integration of equation (A7) from $y^\dagger = 0$ along $\hat{y}$ avoids positive tensile stresses in the region $x^\dagger \in (1/4, 1/2)$. If $\zeta(\tan \beta, \tan \alpha) \geq 0$, consider a region near the free surface. Without wind, the only body force is gravity, so $\beta = +\pi/2$. For weak winds, the surface experiences a small negative $\tau$. Figure 6 maps values of $\tan \beta$ that avoid tensile stresses or, equivalently, where $\zeta(\tan \beta, \tan \alpha) > 0$. To construct this diagram, we assume that the granular assembly retains the state of stress it had when the wind first built it i.e., $\tau < 0$. Consistent with condition (A7), this implies $\tau_c < 0$ for $\tan \alpha < 1/\sqrt{3}$.

Then, because $\zeta > 0$ at $\beta > \pi/2$, $\forall \tan \alpha$, there are no tensile stresses near the surface at low wind, or deep in the bed. As wind strengthens, $\beta$ decreases, and eventually reaches $\beta_0$ such that $\zeta(\tan \beta_0, \tan \alpha) = 0$ (solid lines in Figure 6). Decreasing $\beta$ further would let $\zeta < 0$ and, near enough the surface, this would produce tensile stresses. To avoid these wind strengths further, the assembly must swell to cancel $\zeta$ near the surface, thus maintaining $\sigma_y = 0$ anywhere $\zeta$ would otherwise be negative. In this swolled region, $\beta$ remains fixed at $\beta_0$, and $\nu$ satisfies $\sigma_y = 0$ anywhere. Consider the equation (A7) for $\nu_c = 0$ anywhere $\zeta = 0$.

$$
\frac{\nu}{(1 - \nu)^3} = \frac{\nu_c}{(1 - \nu_c)^3} \times \frac{\cos \beta_0}{R \exp(-2\pi y^\dagger) - \cos(2\pi x^\dagger + \beta_0)}. \quad (A12)
$$

which defines lines of constant $\nu$ throughout the bed. An inspection of this equation reveals that at a fixed $\nu$, such lines exist ($y^\dagger > 0$) in the interval $x^\dagger \in (x^\dagger_c, 1/2)$, where

$$
x^\dagger_c = \frac{1}{2} - \frac{1}{2\pi} \arccos \left[ \frac{\cos \beta_0}{R} \frac{(1 - \nu)^3}{\nu} \frac{\nu_c}{(1 - \nu_c)^3} \right] \frac{\beta_0}{2\pi}. \quad (A13)
$$

The smallest solid volume fractions arise on the free surface near the crest at $x^\dagger = x^\dagger_c \equiv (1 - \beta_0(3\nu_0 / 2\pi))$ if $\beta_0 \leq 0$ or at $x^\dagger_c = 1/2$ otherwise. If the wind speed is too low, $\mathcal{R}$ is small, and drag within the porous bed is not sufficient to swell it, so that $\nu = \nu_c$ everywhere. As the wind strengthens, the first point to swell is at $x^\dagger = x^\dagger_c$. This occurs for the minimum value of $\mathcal{R}$ such that $\nu = \nu_c$ at $x^\dagger = x^\dagger_c$ or, from equation (A13), for $\mathcal{R} = \cos \beta_0$. As $\mathcal{R}$ rises further with increasing wind speed, the line $\nu = \nu_c$ demarcating the swolled region from the underlying compacted bed moves downward, while $\nu$ decreases near the crest.

However, $\nu$ cannot become too small, lest the bed become too dilute to have a solid, interconnected network of contacts. With simulations of hard spheres, Richard et al. [1999] showed that packings below the “melting” transition $\nu < 0.545$ [Zamponi, 2007] cannot form crystals. Thus a granular assembly at $\nu < 0.545$, which lies just below the smallest $\nu$ for minimum fluidization [Wen and Yu, 1966] or random loose packing of spheres [Onoda and Liniger, 1990], hardly transmits stress. In this view, we expect $\nu_m \approx 0.545$ to be the smallest $\nu$ that crests can exhibit. For strong winds, the point $x^\dagger = x^\dagger_c$ near the crest maintain $\nu = \nu_m$ by losing particles to the saltating cloud, while $\mathcal{R}$ remains invariant. The latter is found by substituting $\nu = \nu_m$, $y^\dagger = 0$ and $x^\dagger = x^\dagger_c$ in equation (A11),

$$
\mathcal{R} = \mathcal{R}_c = \frac{\nu_c}{\nu_m} \left( \frac{1 - \nu_m}{1 - \nu_c} \right)^3 \cos \beta_0. \quad (A14)
$$

Figure 7 illustrates the predictions of equation (A11) for $\nu$ for a wind at $\mathcal{R} = \mathcal{R}_c$. Although the corresponding magnitude of $\nu$ at the surface agrees well with the measurements of Figure 3 (dashed line, top graph), the winds required to reach this value of $\mathcal{R}$ are unrealistically strong. For Aijout sand with $\mu_F \approx 0.5$, $\tan \beta_0 \approx 0.33$, and $\rho_s \approx 2500 \text{kg/m}^3$ forming fully-developed ripples with $h_0 / L \approx 0.0195$ and $L \approx 14.5 \text{cm}$, the required wind speed to maintain $\mathcal{R} = \mathcal{R}_c \approx 2.2$ is much too large, $U_{crit} \approx 70 \text{ m/s}$. (We also expect any sand cohesion to raise this velocity further). Therefore, seepage is not responsible alone for variations of $\nu$ at the surface and in the depth.

Note that, even at the largest speed, the Reynolds number $Re_p \equiv \rho u D / \mu$ within the porous bed remains small. Combining equations (9) and (10) at $\nu = \nu_c$ with equation (13), we find that, for $\mathcal{R} < \mathcal{R}_c$, $Re_p < \mathcal{A} \left(1 - \nu_m \right)^3 / (150u_m)$, where $\mathcal{A} \equiv \rho_s \rho g d^2 / \mu$ is the Archimedes number. Then, at worst, $Re_p < 0.9$ in the Sahara, which is smaller than the Reynolds number $\approx 860 \sim 55$ at the transition of a porous bed at $\nu = \nu_c$ from viscous to inertial flow. Although seepage fails to explain the variations of $\nu$ for realistic wind speeds, its swelling model is instructive. First,
seepage counteracts a notable part of gravity even for typical winds. Second, because \( \cos \beta_0 < 1 \) decreases with \( \tan \alpha \), bed friction facilitates such relief, and different sands may have different swelling behavior. Third, the swelling model, which predicts vanishing solids stress where the bed expands, envisages wide regions sustaining little or no stress around and beneath ripple crests (Figure 7). The model also predicts that the depth of these regions is on the order of \( L/(2\pi) \), which is consistent with the intact crests in Figure 4: if crests merely featured a thin region of loose packing near the surface, then they would have erupted as troughs did. Fourth, seepage swelling may be responsible for the disappearance of ripples at very large winds [Bagnold, 1942]. Finally, because the critical swelling wind speed \( U_{cr} \) grows with \( L \), “giant” ripples should survive greater wind speeds.

**Appendix B: Free-surface granular shear flow**

We consider a thin shear layer of agitated grains sketched in the bottom of Fig. 7. It travels on an erodible base, is driven by a shear stress \( \tau_0 \) applied on its free surface, and borrows agitation at that surface from turbulent gas velocity fluctuations. Because typical rippled surfaces make a small angle \( < \arctan(2\pi H_0/L) \approx \pi^2 \) to the horizontal, we neglect gravity along \( x \), so that grains are moved by \( \tau_0 \) alone. Because the thin shear layer has relatively dense particle loading, its grains entrain the surrounding air at their own velocity. Thus, we ignore drag, making our analysis simpler than in the dependent variables \( \tau, \gamma_0, \gamma_1 \) and \( \sigma_y \). We deduce \( \nu \) from \( -\sigma_y/\langle \rho \nu \rangle \) in equation (B4) by inverting \( f_4(\nu) \) using a look-up table. On the free surface at \( y = 0 \), we prescribe the shear stress \( \tau = \tau_0 < 0 \) and invoke the boundary conditions of Jenkins and Hanes [1993] relating the normal stress

\[
\sigma_y = -2
\]

and the temperature \( T_0 \). There, the volume fraction satisfies \( \nu_0 \tau_1 = 1/4 \) or \( \nu_0 \simeq 0.16 \). For a horizontal free surface, Jenkins and Hanes [1993] also prescribed \( q = 0 \) at \( y = 0 \).

Parameters of this problem are \( \gamma_0, T_0, \tau_0, \) and \( \epsilon \), in which \( \epsilon \) is an effective parameter characterizing energy lost in the impacts of two inelastic, frictional grains. Jenkins and Zhang [2002] calculated \( \epsilon \) in terms of the kinematic coefficient of normal restitution \( e_N \) and the Coulomb friction coefficient \( \mu_f \). To represent grains of sand, we adopt \( \epsilon \) and \( \mu_f \) as described in [Lorenz et al., 1997].

We relate temperature and shear stress at the free surface by invoking Pourahmadi and Humphrey [1983], who wrote \( T = 2k/(1 + St) \), where \( St \) is the Stokes number, \( k \equiv u^2_s + u^2_y + u^2_z \), where \( v_i^2 \equiv \bar{v}_i - v_i \), \( v_i \) is the instantaneous gas velocity, \( v_i \) is its average over time and \( i = x, y, z \). The local fluctuation energy balance (resembling the k-equation in turbulence) is

\[
-\frac{\partial q/\partial y + \tau \partial v/\partial y - \gamma = 0,}
\]

where the flux of fluctuation energy is

\[
q = -f_3(\nu, \nu) \rho \nu T^{1/2} \partial v/\partial y,}
\]

in which \( f_3(\nu, \rho \nu) \) is a granular conductivity, and the volume rate of dissipation of fluctuation energy (resembling the turbulent energy dissipation rate \( \epsilon \)) is

\[
\gamma = f_3(\nu, \nu) \rho \nu T^{3/2}/d.
\]

We eliminate the velocity derivative in equation (B5) using equation (B3) and write

\[
\frac{\partial q/\partial y = \pi^2/[\nu_1 \rho \nu T^{1/2}] - \gamma.}
\]

For nearly elastic, frictionless spheres of restitution coefficient \( e \), Jenkins and Richman [1985] calculated

\[
f_1 = \frac{8}{5\pi} \nu^2 \tau_1 \pi^2 \left[ \frac{1 + \pi}{12} \left( \frac{5}{8\nu \tau_1} \right)^2 \right],
\]

\[
f_2 = \frac{4}{\pi} \nu^2 \tau_1 \pi^2 \left[ \frac{1 + \epsilon}{3} \left( \frac{5}{12\nu \tau_1} \right)^2 \right],
\]

\[
f_3 = \frac{12}{\pi} \nu^2 \tau_1 \pi^2 \left[ \frac{1 - \epsilon^2}{1 - \epsilon^2} \right],
\]

and

\[
f_4 = \nu^2 \left( 1 + \nu \tau_1 \right),
\]

where \( \tau_1/\nu \) is the pair distribution of colliding grains “1” and “2” at contact, which, for two identical spheres, is well represented by the Carnahan and Starling [1969] expression

\[
\tau_1 = \frac{2 - \nu}{2(1 - \nu^3)}.
\]

Equations (B1), (B2), (B6) and (B8) are non-linear ODEs in the dependent variables \( \tau, \gamma_0, \gamma_1 \) and \( \sigma_y \). We deduce \( \nu \) from \( -\sigma_y/\langle \rho \nu \rangle \) in equation (B4) by inverting \( f_4(\nu) \) using a look-up table. On the free surface at \( y = 0 \), we prescribe the shear stress \( \tau = \tau_0 < 0 \) and invoke the boundary conditions of Jenkins and Hanes [1993] relating the normal stress

\[
\sigma_y = -2
\]

and the temperature \( T_0 \). There, the volume fraction satisfies \( \nu_0 \tau_1 = 1/4 \) or \( \nu_0 \simeq 0.16 \). For a horizontal free surface, Jenkins and Hanes [1993] also prescribed \( q = 0 \) at \( y = 0 \).

Parameters of this problem are \( \gamma_0, T_0, \tau_0, \) and \( \epsilon \), in which \( \epsilon \) is an effective parameter characterizing energy lost in the impacts of two inelastic, frictional grains. Jenkins and Zhang [2002] calculated \( \epsilon \) in terms of the kinematic coefficient of normal restitution \( e_N \) and the Coulomb friction coefficient \( \mu_f \). To represent grains of sand, we adopt \( \epsilon \) and \( \mu_f \) as described in [Lorenz et al., 1997].

We relate temperature and shear stress at the free surface by invoking Pourahmadi and Humphrey [1983], who wrote \( T = 2k/(1 + St) \), where \( St \) is the Stokes number, \( k \equiv u^2_s + u^2_y + u^2_z \), where \( v_i^2 \equiv \bar{v}_i - v_i \), \( v_i \) is the instantaneous gas velocity, \( v_i \) is its average over time and \( i = x, y, z \). The local fluctuation energy balance (resembling the k-equation in turbulence) is

\[
-\frac{\partial q/\partial y + \tau \partial v/\partial y - \gamma = 0,}
\]

where the flux of fluctuation energy is

\[
q = -f_3(\nu, \nu) \rho \nu T^{1/2} \partial v/\partial y,}
\]

in which \( f_3(\nu, \rho \nu) \) is a granular conductivity, and the volume rate of dissipation of fluctuation energy (resembling the turbulent energy dissipation rate \( \epsilon \)) is

\[
\gamma = f_3(\nu, \nu) \rho \nu T^{3/2}/d.
\]
solids [Kulick et al., 1994]. We adopt the peak value of 

\[ C_r \approx 0.25 \] at the wall [Patel et al., 1985].

Combining these expressions, we find

\[ T_0 \sim \left( \frac{2R_{\rho}}{3C_T^{1/2}} \right) \frac{|\tau_0|/\rho_s}{(1 + \text{St})}, \]

(B16)

where \( R_{\rho} \equiv \rho_s/\rho \) is the material density ratio and

\[ \text{St} = \frac{R_{\rho}}{C_T} \left( \frac{C_T^{1/2} C_r \text{Ar}}{18} \frac{|\tau_0|/(\rho_s g d)}{1 + 0.15 \text{Re}^0.987} \right) + 2 \left( \frac{\nu_0}{1 - \tau_0} \right). \]

(B17)

For typical conditions in the Sahara, \( \text{St} \geq 5700 \gg 1 \), justifying our implicit assumption in equation (B5) that granular agitation within the shear layer is unaffected by the gas.

We solve the four ODEs from the free surface at \( y = 0 \) downward along the \( y \)-axis to the passive base using a fourth-order Runge-Kutta procedure until the granular agitation (B5) that granular shear layer obeys is inhibited by the gas.

We evolve the four ODEs that govern the free shear layer at \( y = 0 \) downward along the \( y \)-axis to the passive base using a fourth-order Runge-Kutta procedure until the granular agitation (B5) that granular shear layer obeys is inhibited by the gas.

Agitated shear layers do not exist at arbitrarily small \( |\tau_0| \).

Because they occur on an erodible bed that only dissipates fluctuation kinetic energy [Jenkins and Askari, 1991], their agitation is sustained if and only if the volumetric production rate of fluctuation rate of energy in the second term of equation (B5) exceeds the volumetric dissipation rate \( \gamma_s \). Thus, as \( |\tau_0| \) decreases, the shear layer loses its agitation and eventually collapses at a minimum surface shear stress \( |\tau_{0crst}| \) below which equations (B1)-(B17) have no solution. By solving the numerical model for a number of conditions within which equations (B1)-(B17) have no solution, we find that, locally, the minimum shear velocity \( \nu_{s,\text{crit}} \) necessary to create an agitated granular shear layer obeys approximately

\[ \rho_{s,\text{crit}}^2 \equiv -\tau_{0crst} \approx \rho_s g d \left( 0.09 + 0.135 (1 - e^{-2})^{0.4} \times \exp\left(-\left(\frac{\text{Ar}/\text{Ar}_0}{1/2}\right)\right) \right), \]

(B18)

where \( A \approx 6000 \). Although this result overestimates by a factor of 2.7 the threshold shear velocity that Shao and Lu [2000] correlated from empirical data, it predicts, for example, how Martian winds must be faster than Earth’s to reach the particle entrainment threshold.

This model also yields the average volume fraction of grains in the shear layer

\[ \bar{\nu} \equiv \frac{1}{h} \int_{y=0}^{h} \nu dy, \]

(B19)

in which we exploit equation (B2) to evaluate the integral,

\[ \bar{\nu} = -[\sigma_y(y = h_t) - \sigma_y(0)]/\rho_s h_t. \]

(B20)

The result is conveniently represented by

\[ \bar{\nu} = \nu_{s,\infty} + (\nu_s - \nu_{s,\infty}) \exp\left(-|\tau_0 - \tau_{0crst}|/\tau_{0crst}\right) \]

(B21)

with \( \nu_{s,\infty} \approx 0.27 \), \( -\tau_{0crst}/(\rho_s g d) \approx 0.17 \), and \( \tau_{0crst} \) from equation (B17). This shows how the granular shear layer becomes less dense as its mobility increases.

Because it ignores the role of saltation and “splash” in setting agitation \( T_0 \) and flux \( q \) at the free surface, this “toy model” of the shear layer can only describe qualitatively the flow of reptating grains. Because its excludes grain interactions other than binary collisions, it likely fails near the threshold. However, the model indicates that, because \( |\tau_0| \) peaks at crests, reptation is sustained at lower wind speeds on crests compared to troughs. Its predictions for the altitude \( T_0/q \) to which grains are ejected above the free surface also yields a simple closed-form expression of the saltation mass flow rate [Louge et al., 2009b] that agrees well with correlations [Sørensen, 2004].

Finally, it is instructive to relate our calculations of normal stress at the free surface to the predictions of Creyssels et al. [2009], who recently reported how saltation interacts with the sand bed at steady state. Upon balancing the granular influx hitting the ground and the subsequent outflux of ejecta, these authors showed that saltating grains have a granular temperature aloft that is invariant with \( u^* \) and is empirically set by dimensionless impact parameters of the splash, which do not depend on wind strength: because fast-hitting grains produce more ejecta, while slower grains remain trapped, the velocity variance of grains in saltation evolves until there is no net imbalance between influx and outflux. Creyssels et al. [2009] predicted the volume fraction \( \nu_{s,\text{salt}} \) of grains in saltation

\[ \nu_{s,\text{salt}} \approx \frac{1}{\mu_{sp}} \exp \left(-\frac{gh}{T_{s,\text{salt}}}(\tau_0 - \tau_{0th})/\rho_s T_{s,\text{salt}} \right), \]

(B22)

which is independent of \( T_{s,\text{salt}} \). Returning to the granular shear layer, because \( \text{St} \gg 1 \), there are two limits for \( T_0 \), depending whether solids or gas turbulent dissipation dominate. If \( \epsilon_g \gg \epsilon_s \), then \( T_0/(\rho_s g d) \approx 2C_T (1 + 0.15 \text{Re}^{0.987})/(C_r C_s \text{Ar}) \) varies weakly with \( \tau_0 \), but strongly with particle size through \( \text{Ar} \). This limit arises at low solid volume fraction; it is reminiscent of the nearly invariant temperature that Creyssels et al. [2009] observed in saltation, although these authors invoked splash, – rather than particle-turbulence interactions –, to evaluate it. Instead if \( \epsilon_s \gg \epsilon_g \), then \( T_0 \approx |\tau_0|/\rho_s g d/3C_{p,\text{salt}} \) grows linearly with \( \tau_0 \), but is independent of \( \text{Ar} \). Because \( \nu_0 \approx 0.16 \) is relatively high, this limit typically dominates in the shear layer. In this case,

\[ -\sigma_y \approx 2 \mu_{sp} T_0 \approx 2C_T (1 + \nu_0)/(3C_{p,\text{salt}}/2) \approx 0.8|\tau_0|, \]

which compares well with equation (B23) derived from Creyssels et al. [2009] at wind speeds sufficiently above threshold.

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