An experimental verification of Glicksman's reduced scaling

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ABSTRACT
A dimensional analysis of the momentum balance in circulating fluidized beds reveals that the fluid dynamics of these units is governed by five dimensionless groups. However, it is not possible to match the hydrodynamics of a range of industrial circulating fluidized beds using a single laboratory facility operated at ambient temperature and pressure.

To address this difficulty, Glicksman, et al. (1993) proposed a "reduced set" involving only four dimensionless parameters. They showed that the "reduced set" is complete when the drag on solid particles conforms either to the viscous or to the inertial limit. They assumed that it would also succeed in the intermediate regime, where many industrial beds operate.

We verify this assumption by comparing two series of experimental data obtained in the same facility with different combinations of gases and solids that preserve the "reduced set" but exhibit a mismatch of the complete five-parameter set of dimensionless numbers. We find that the dimensionless profiles of pressure along the flow and solid volume fraction in the radial direction are nearly identical in the two series of experiments. These observations validate the reduced scaling proposed by Glicksman et al. for the intermediate regime, at least for the simulated pressurized conditions of our experiments.

INTRODUCTION
The hydrodynamic scaling of circulating fluidized beds (CFB) remains an industrial challenge. In an effort to increase process efficiency, a recent trend in coal combustion technology has been to pressurize CFB units. Under these conditions, studies of hydrodynamic scale-up are limited. One approach is to construct a smaller pilot facility of similar geometrical aspect ratios fluidized with inert gas mixtures of adjustable density at ambient pressure and temperature. The five dimensionless groups governing the flow in the pilot are then matched to those in the industrial unit to achieve hydrodynamic similarity (Louge, et al., 1999). A difficulty with this method is that the matching imposes onerous constraints on the size of the pilot and properties of the gases and solids.

To address this difficulty, Glicksman, et al. (1993) proposed a simpler approach that relaxes these constraints. They noticed that, in the viscous and inertial limits of the drag on individual solid particles, the momentum balances of the two phases reduce to a form giving rise to only four dimensionless groups. They then postulated that this “reduced set” of parameters would also govern the fluid dynamics of suspensions operating...
in the intermediate regime. Our objective is to check this assumption for CFBs operating under pressure.

We begin with a summary of the simpler approach. We then briefly describe our experiments and discuss their results.

**REDUCED SCALING**

The commercialization of large pressurized coal combustion units is inhibited by the uncertain extrapolation from a pilot plant to full size industrial units. In this context, dimensional analysis, commonly used in wind tunnels or flumes, is a powerful tool for testing a design on a smaller scale. In the absence of inter-particle forces or electrostatics, continuum equations for gas-solid suspensions derived, for example, by Anderson and Jackson (1967) yield five dimensionless groups

\[
Fr = \frac{u}{\sqrt{g \phi d_s}}, \quad \text{(1)}
\]

\[
M = \frac{G}{\rho u}, \quad \text{(2)}
\]

\[
Ar = \frac{\rho_s \rho (\phi d_s)^3}{g^2}, \quad \text{(3)}
\]

\[
R = \frac{\rho_s}{\rho}, \quad \text{(4)}
\]

\[
L = \frac{D}{\phi d_s}, \quad \text{(5)}
\]

where \(\rho\), \(\mu\), and \(\rho_s\) are the density of the gas, its viscosity, and the material density of the solids, respectively; \(u\) is the superficial gas velocity; \(G\) is the overall solid flux; \(g\) is the acceleration of gravity; and \(D\) is the riser diameter.

The Froude number \(Fr\) and solid loading \(M\) are the operational parameters. The first is a dimensionless measure of the superficial gas velocity. The second is the ratio of the overall solid and gas fluxes. The Archimedes number \(Ar\) combines gas and solid properties. It arises naturally in correlations capturing the terminal velocity of an individual particle. \(R\) is the density ratio. The dimensionless size \(L\) is a measure of the scale of the unit. Following the suggestion of Chang and Louge (1992), the effective particle diameter is the product of the mean particle Sauter diameter \(d_s\) and its sphericity \(\phi\). Other dimensionless aspect ratios such as the ratio of riser height and diameter and the particle size distribution relative to \(d_s\) must also be matched in the scaling.

Because they require a new cold model to simulate each new industrial unit, the set of five scaling parameters is impractical. This predicament has led Glicksman et al. (1993) to seek a simpler alternative. These authors showed that scaling can be studied from a balance of forces on a particle entrained in the riser, and focused their attention on the drag term.

For illustration purposes, let us focus on the solid momentum balance and assume, for simplicity, that shear stresses are negligible and that the flow is steady and one-dimensional. The resulting momentum equation is

\[
\rho_s (1-\varepsilon) \frac{dv}{dz} = - (1-\varepsilon) \frac{dp}{dz} - \rho_s (1-\varepsilon) g + F, \quad \text{(6)}
\]

where \(\varepsilon\), \(v\) and \(p\) are the voidage, solid velocity and static gas pressure, respectively; \(z\) is the elevation and \(F\) is the drag force per unit volume exerted by the gas. In the viscous limit, this force has the form
\[ F = \frac{18 \mu}{d_s^2} u_{\text{slip}} (1-\varepsilon) , \]  

where \( u_{\text{slip}} \) is the local slip velocity. In the inertial limit, it is

\[ F = \frac{3\rho}{4d_s} C_d u_{\text{slip}} (1-\varepsilon) , \]

where the drag coefficient \( C_d \) is approximately constant. To eliminate \( \mu \) in the viscous limit and \( C_d \) in the inertial limit, we use the balance of weight and drag that determines the terminal velocity \( u_t \) and assume that \( u_{\text{slip}} \) and \( u_t \) are on the same order. We can then write the drag force as

\[ F = \rho_s g \left( \frac{u_{\text{slip}}}{u_t} \right)^n (1-\varepsilon) , \]

where the exponent \( n \) is unity in the viscous limit and two in the inertial limit.

Upon making the momentum balance dimensionless by scaling lengths, velocities and pressure with \( D, u \) and \( \rho u^2 \), respectively, we obtain

\[ (1-\varepsilon) v^* \frac{dv^*}{dz^*} = - \left( \frac{\rho}{\rho_s} \right) (1-\varepsilon) \frac{dp^*}{dz^*} - \left( \frac{gD}{u^2} \right) (1-\varepsilon) + \left( \frac{gD}{u^2} \right) u_{\text{slip}}^* \left( \frac{u}{u_t} \right)^n , \]

where asterisks denote dimensionless quantities. A dimensionless equation with similar parameter combinations arises from the gas momentum balance. Consequently, hydrodynamic similitude only requires the matching of four dimensionless numbers, namely

\[ R = \frac{\rho_s}{\rho} , \]
\[ Fr^2/L = \frac{u^2}{gD} , \]
\[ V = \frac{u}{u_t} \]

and, as before, a measure of the overall solid loading in the riser, which for convenience we choose to express as the ratio

\[ M/R = \frac{G}{\rho_s u} . \]

In short, Glicksman, et al. (1993) simplify the similitude by scaling the drag term with the terminal velocity. In doing so, they assume that the mean slip velocity experienced by the solid phase is proportional to \( u_t \). In addition, to extend their approach in between the two limits, they show that the simpler formulation of Eq. (10) introduces negligible errors in the drag in the intermediate regime.

A practical difficulty is that one must know \( u_t \) to employ this method. Arguing that it is easier to measure the minimum fluidization velocity \( u_{mf} \) than the terminal velocity, and that, in the absence of inter-particle forces the ratio of these two velocities is roughly constant in the viscous and inertial limits, Glicksman, et al (1993) suggested that \( u_{mf} \) be substituted for \( u_t \) in Eq. (12). In this study, because the determination of \( u_{mf} \) is complicated by inter-particle surface forces not relevant to the physics of entrained suspensions, we choose to follow Glicksman’s original intent and base the similitude on terminal velocity. Unfortunately, for a powder of significant particle size distribution, it is difficult to measure \( u_t \). Instead, we rely on the correlation of Haider and Levenspiel (1989) for spherical and non-spherical particles.
To test this approach in the intermediate regime, we performed two sets of experiments with different materials and gases that matched all four parameters of the reduced set (Eqs. 4, 11, 12, 13), but not all five parameters of the full set (Eqs. 1 to 5). We could have achieved this by mismatching Ar, L or Fr, or any of their combination. However, for convenience, we chose to mismatch Ar while preserving L and Fr, which allowed us to test the reduced theory with the same mixtures that Louge et al. (1999) employed in their study of pressurized CFBs.

EXPERIMENTS

The circulating fluidized bed facility is sketched in Fig. 1. It includes a riser of 0.2 m diameter and 7 m height running near ambient temperature and pressure. Its unique ability is to recycle any mixture of inert fluidization gases, thus setting all five dimensionless numbers at the desired values. Louge, et al. (1999) provided details of its operations.

![Fig. 1. The circulating fluidized bed facility.](image)

We validated the concept of reduced scaling in two ways. First, we acquired vertical profiles of static pressure along the riser using 35 ports connected to a single scanning valve and pressure transducer. We compared the profiles in dimensionless form as \( p^\dagger = (p-p_{\text{top}})/(\rho_s g D) \) versus relative elevation \( z^\dagger \equiv z/H = z^* (D/H) \), where \( p_{\text{top}} \) is the static pressure at the top and \( H \) is the riser height.

Second, we recorded radial profiles of solid volume fraction using an instrument that combines a capacitance probe inserted flush with the wall and a probe featuring a random bundle of optical fibers. The optical probe was calibrated against the quantitative capacitance probe at the wall. It was then traversed across the riser (Beaud and Louge, 1995).

Experimental conditions are summarized in Table 1 and Fig. 2. We carried out two sets of experiments to simulate a generic pressurized coal-burning CFB operating at 1200 °K and 0.81 MPa with solids of material density \( \rho_s = 1.5 \text{ g/cm}^3 \), mean Sauter diameter \( d_s = 400 \mu\text{m} \) and sphericity \( \phi = 0.7 \). Tests were conducted with the ratio \( u/u_t \) in the range \( 5 \leq V \leq 14 \). The two sets shared the same reduced set of dimensionless groups, but differed in the Archimedes number. The corresponding glass and plastic powders conformed to the same generic relative particle size distribution that Chang and Louge (1992) and Louge, et al. (1999) used. Consequently, as Chang and Louge (1992)
showed by analyzing the Haider and Levenspiel correlation for $u_t$, the resulting distribution $u_t(d_i)/u_t(d_s)$ of the terminal velocities of each size class $d_i$ relative to the mean was nearly identical for the two powders. (For illustration purposes, we measured $u_{mf} = 0.91 \text{ cm/s}$ and 1.03 cm/s in the downcomer for the glass and plastic powders, respectively). Table 1 also provides the range of Reynolds numbers based on $u_{slip}$ for each experiment,

$$Re_{slip} = u_{slip} \frac{\rho_s d_s}{\mu} = \lambda Fr \sqrt{Ar/R} ,$$

which we evaluate using the slip coefficient $\lambda$ that Louge, et al. (1999) measured. As intended, the corresponding values of $Re_{slip}$ are neither very large nor very small.

<table>
<thead>
<tr>
<th>Fluidization gases</th>
<th>Solid powders</th>
<th>Analogous coal unit</th>
<th>Dimensionless numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF6 % Air %</td>
<td>$\rho$ kg/m$^3$</td>
<td>$\mu \times 10^5$ kg/m.s</td>
<td>type</td>
</tr>
<tr>
<td>SF6</td>
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</tr>
<tr>
<td>SF6</td>
<td>24</td>
<td>76</td>
<td>2.37</td>
</tr>
</tbody>
</table>

Fig. 2. Operational conditions for the experiments with glass beads (circles) and plastic grit (diamonds).

**RESULTS**

As Louge, et al. (1999) reported, the vertical pressure profiles depended strongly on solid loading, but weakly on superficial gas velocity. The relatively small extent of their acceleration region was typical of pressurized conditions. As Fig. 3 illustrates, the tests with plastic and glass powders exhibited the same vertical pressure profiles in the upper region of the riser, thus confirming an insensitivity to the Archimedes number there, and verifying the validity of the reduced scaling when the flow is nearly fully-developed.

However, pressure profiles corresponding to the largest values of loading in the plastic powder exhibited a higher acceleration region than in the glass powder. This discrepancy, which arose mainly around the inlet of solids returning from the downcomer at an elevation in the range $0.06 < z^\dagger < 0.10$, coincided with a difficulty to establish stable recirculation of plastic grit through the riser. Under those conditions, the inventory
of plastic powder in the downcomer was much higher than its glass counterpart. These observations suggest that limitations of the reduced set approach are associated with conditions where recirculation instabilities develop from the interaction dynamics between downcomer and riser.

Perhaps because radial profiles of solid volume fraction were recorded at an elevation \( z^+ = 0.51 \) within the fully-developed region, we observed an excellent match between the profiles measured with glass and plastic powders under all operating conditions (Fig. 4). However, despite this remarkable matching, the solid volume fraction recorded with the capacitance probe at the wall was usually higher in plastic than in glass. This discrepancy may result from slight differences in the detailed mechanical properties of the two materials, which are not taken into account in the reduced scaling.

Fig. 3. Dimensionless vertical profiles of static pressure for \( \text{Fr}^2/L = 5.2 \) and the solid loadings \( M/R = 0.0016 \) (left), \( 0.0043 \) (middle) and \( 0.0075 \) (right). Circles and diamonds represent experiments with glass beads and plastic grit, respectively.

Fig. 4. Profiles of solid volume fraction along the radial coordinate \( r \) for \( \text{Fr}^2/L = 5.2 \) and the solid loadings \( M/R = 0.0016 \) (bottom) and \( 0.0075 \) (top). The circles and diamonds represent experiments with glass beads and plastic grit, respectively. The lines are visual fits through the data.
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REFERENCES


